University of Swaziland

Final Examination, December 2016

B.A.S.S. , B.Sc, B.Eng, B.Ed

Title of Paper : Numerical Analysis I

Course Number : M311

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: Answer All Questions

A1. (a) i. Convert the decimal number $\frac{43}{5}$ to its binary equivalent. [4]

ii. Given the function $f(x) = -3 + \sqrt{9-x}$, find a suitable function $g(x)$ that has been reformulated to be algebraically equivalent to $f(x)$ with the aim of avoiding loss of significance error. [3]

(b) i. Estimate the root of the equation, $\ln x - e^x + 3 = 0$ using 3 iterations of the **Newton Method** with starting point $x_0 = 1$. [6]

ii. Give one advantage of the Newton Method. [1]

(c) i. State the Weierstrass Approximation Theorem. [3]

ii. Given the nodes, $x_0 = 2,\ x_1 = 2.75$ and $x_2 = 4$, generate the cardinal functions associated with the Lagrange interpolating polynomial of second order. [6]

(d) i. Give the general formula for the nth divided difference $f[x_0, x_1, \ldots, x_n]$. [3]

ii. Given the set of data points $\{(0,1), (2,5), (4,17)\}$, generate Newton's interpolation polynomial of degree 2 which passes through these points. [6]

(e) Find the approximate value of

$$I = \int_0^1 \frac{dx}{1+x},$$

using the Trapezoidal rule. [3]

(f) Given the following linear system of equations, formulate the Gauss-Seidel iterative scheme for the k^{th} approximate solution.
DO NOT SOLVE.

$$\begin{array}{rcrcrcr} 5x_1 & - & 2x_2 & + & 3x_3 & = & -1, \\ -3x_1 & + & 9x_2 & + & x_3 & = & 2, \\ 2x_1 & - & x_2 & - & 7x_3 & = & 3. \end{array}$$

[5]

SECTION B: Answer Any 3 Questions

B2. (a) Convert the decimal 5.125 into its binary equivalent. [6]

(b) Convert the binary $(0.\overline{101})_2$ into its decimal equivalent. [6]

(c) Determine the machine representation in single precision on a 32-bit word length computer for the decimal number -12.75. [8]

B3. Consider the function $f(x) = x^3 + 4x^2 - 10$.

(a) Show that $f(x)$ has exactly one root in $[1,\ 2]$. [6]

(b) By performing 4 iterations of the **Bisection Method**, show that this root lies in the interval $[1.3125,\ 1.375]$. [8]

(c) How many iterations would be required to locate this zero to a tolerance of 10^{-5} ? [4]

(d) Give two advantages of the Bisection Method. [2]

B4. (a) Consider the following $(n+1)$ data points.

x	x_0	x_1	$\dots$	x_n
$f(x)$	$f(x_0)$	$f(x_1)$	$\dots$	$f(x_n)$

where $x_0 < x_1 < x_2 < \cdots < x_n$. Write down the Lagrange and Newton interpolation polynomials respectively for the given data. [6]

(b) Find the Lagrange interpolation polynomial for the following data.

x	0	2	3
$f(x)$	7	11	28

[8]

(c) Consider the points $x_0 = 0$, $x_1 = 0.4$, $x_2 = 0.7$, and for a function $f(x)$, the divided differences are $f[x_2] = 6$, $f[x_1, x_2] = 10$, $f[x_0, x_1, x_2] = 50/7$. Use this information to construct the complete divided difference table for the given points. [6]

B5. (a) Use the Composite Trapezoidal rule with $n = 4$ to estimate

$$\int_1^3 \sqrt{1+x^2}dx.$$

[6]

(b) Determine the values of h and n that will ensure that the error of the Composite Simpson's rule to compute the integral $\int_0^1 e^{-2x}dx$ is less than 10^{-5}. [6]

(c) For the following observation data,

x	-1	0	1
$f(x)$	$f(-1)$	$f(0)$	$f(1)$

find the corresponding Newton-Cotes quadrature rule for approximating the integral $\int_{-2}^2 f(x)dx$. [8]

B6. (a) Use Gaussian elimination to solve the system of linear equations.

$$\begin{aligned} 2x_2 + x_3 &= -8, \\ x_1 - 2x_2 - 3x_3 &= 0, \\ -x_1 + x_2 + 2x_3 &= 3. \end{aligned}$$

[10]

(b) Consider the linear system,

$$\begin{aligned} 4x_1 + x_2 &= 1, \\ x_1 + 4x_2 - x_3 &= 2, \\ x_2 + 4x_3 &= 3. \end{aligned}$$

Starting with $X^{(0)} = [0, 0, 0]^T$, use the Gauss-Seidel iterative method to find $X^{(1)}$. [10]

END OF EXAMINATION