University of Swaziland

Supplementary Examination, July 2017

B.A.S.S. , B.Sc, B.Eng, B.Ed

Title of Paper : Numerical Analysis I

Course Number : M311

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.

   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.

   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.

2. Each question in Section B is worth 20%.

3. Show all your working.

4. Non programmable calculators may be used (unless otherwise stated).

5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: Answer All Questions

A1. (a) i. Convert the binary number $(0.\overline{101})_2$ to its decimal equivalent. [3]

ii. How can the function $f(x) = e^x - x - 1$, near $x = 0$ be re-written to avoid loss of significance error? [3]

(b) i. Estimate the root of the equation, $\ln x - e^x + 3 = 0$ using 3 iterations of the **Secant Method** with starting points $x_0 = 1$ and $x_1 = 2$. [6]

ii. Give one advantage of the Secant method over the Newton method. [1]

(c) Consider the following $(n+1)$ data points.

x	x_0	x_1	$\dots$	x_n
$f(x)$	$f(x_0)$	$f(x_1)$	$\dots$	$f(x_n)$

where $x_0 < x_1 < x_2 < \cdots < x_n$. Write down the Lagrange and Newton interpolation polynomials respectively for the given data. [6]

(d) i. Use the linear Lagrange polynomial

$$P_1(x) = \frac{(x - x_1)}{(x_0 - x_1)} f(x_0) + \frac{(x - x_0)}{(x_1 - x_0)} f(x_1),$$

to derive the Trapezoidal rule for approximating $\int_a^b f(x)dx$. [6]

ii. Approximate $\int_0^2 \frac{1}{x+1}$ using the "basic" Simpson's rule. Compare your results with the exact value. [4]

(e) Consider the following linear system of equations.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 9, \\ 2x_1 - x_2 + x_3 &= 8, \\ 3x_1 \qquad\quad - x_3 &= 3. \end{aligned}$$

i. Formulate the Jacobi iterative scheme for the k^{th} approximate solution. DO NOT SOLVE. [3]

ii. Solve the system using the Gaussian elimination method. [8]

SECTION B: Answer Any 3 Questions

B2. (a) Convert the decimal 23.3125 into its binary equivalent. [6]

(b) Convert the binary 1101.101 into its decimal equivalent. [6]

(c) Determine the decimal number that has

1 10000010 11110110000000000000000

as its Marc-32 representation. [8]

B3. (a) Consider the function $f(x) = x - \cos x$. Prove that $f(x)$ has exactly one root in $[0, \pi/2]$. [6]

(b) Use 3 iterations of the Newton's method to approximate the zeros of $f(x) = e^x - x^2$ on $[-1, 0]$ accurately using $x_0 = 0$ as initial data. [8]

(c) By using the Bisection method, how many iterations would be required to locate this zero (in (b) above) to a tolerance of 10^{-4} ? [6]

B4. (a) i. Interpolate the table

x	-0.5	0	0.5
$f(x)$	0.146	0.169	0.202

using a suitable polynomial in Newton form. [8]

ii. Use your previous result to approximate $f(0.4)$. [2]

(b) Interpolate the table

x	-1	0	1	2
$f(x)$	5	1	1	11

with a suitable polynomial in Lagrange form. [10]

B5. (a) Use the Composite Trapezoidal rule with $n = 4$ to estimate $\int_1^3 \sqrt{1+x^2}dx$. [6]

(b) Determine the values of h and n that will ensure that the error of the Composite Simpson's rule to compute the integral $\int_0^1 e^{-2x}dx$ is less than 10^{-5}. [6]

(c) For the following observation data,

x	-1	0	1
$f(x)$	$f(-1)$	$f(0)$	$f(1)$

find the corresponding Newton-Cotes quadrature rule for approximating the integral $\int_a^b f(x)dx$. [8]

B6. (a) Solve the linear system

$$\begin{aligned} 2x_1 + 6x_2 + 4x_3 &= -2, \\ x_1 - 2x_2 &= 4, \\ 4x_2 + x_3 &= 2. \end{aligned}$$

using **Cramer's** rule. [10]

(b) Consider the linear system,

$$\begin{aligned} 4x_1 + x_2 &= 1, \\ x_1 + 4x_2 - x_3 &= 2, \\ x_2 + 4x_3 &= 3. \end{aligned}$$

Starting with $X^{(0)} = [0, 0, 0]^T$, use the Jacobi iterative method to find $X^{(2)}$. [10]

END OF EXAMINATION