UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION, 2016/2017

B.Sc./ B.Ed./ BASS III

Title of Paper : Dymanics II

Course Number : M355

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.
   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked.**
2. Show all your working.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

A1. Give definition and example of

(a) degree of freedom, (2)

(b) generalized coordinate, (2)

(c) scleronomic system, (2)

(d) conservative system (2)

(e) transformation equation (2)

A.2 A particle of mass M is moving down in the field of gravity.

(a) Derive Lagrange's equation, (2)

(b) Solve it, (2)

(c) Introduce generalized momentum, (2)

(d) Construct Hamiltonian, (2)

(e) Derive Hamilston's equations (2)

A3. For a certain system the kinetic energy T and potential energy Π are given by $2T = ml^2(\dot{x}^2 + \dot{y}^2 \sin^2 x)$, $\quad \Pi = -mgl \cos x$.

Show that coordinate y is cyclic. (3)

A4. Give at least two conditions for canonical transformation. (3)

A5. Consider a dynamic variable $A(q, p, t)$ and let $H(q, p, t)$ be a Hamiltonian of a system. Prove that

$\dfrac{dA}{dt} = \dfrac{\partial A}{\partial t} + [A, H]$ in the usual notations. (4)

A6. Define,

(a) functional,

(b) variation of a function. (2,2)

A7. Find the extremals of

$$v[y(x)] = \int_0^{\frac{\pi}{2}} (y')^2 - y^2]dx, \qquad y(0) \text{ is free}, \quad y(\tfrac{\pi}{2}) = 0 \qquad (4)$$

A8. For the functional

$$v[y_1, y_2, \cdots, y_n] = \int_{x_0}^{x_1} F(x, y_1, y_2, \cdots, y_n y_1', y_2', \cdots, y_n')dx$$

write Euler's equations. (2)

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B1 [*20 Marks*]

B1. (a) Prove the Interchange of d and ∂ Lemma

$$\frac{d}{dt}\left(\frac{\partial \bar{r}_\nu}{\partial q_i}\right) = \frac{\partial \dot{\bar{r}}_\nu}{\partial q_i}. \qquad (6)$$

(b) Given Lagrange's equations (L.E.) in general case. Derive $L.E.$ for conservative system. (4)

(c) Two particles of mass m_1 and m_2 are connected by a light inextensible string of length l which passes through a smooth hole in a smooth horizontal table. The mass m_2 lies on the table at a distance r from the hole. A gravitation field g acts on this system. If the mass m_1 moves only in vertical line, find the equation of motion for both masses. (4)

(d) The Lagrangian for a certain system is given by

$$L = \frac{m}{2}((\dot{x} - wy)^2 + (\dot{y} + wx)^2 - \Pi(x, y)$$

Write down Lagrange's equations. (6)

QUESTION B2 [*20 Marks*]

B2. (a) Prove that for holonomic, scleronomic system the following formula is valid

$$\sum_{i=1}^{n} \dot{q}_i \frac{\partial T}{\partial \dot{q}_i} = 2T. \qquad (10)$$

(b) Derive the equation of motion for the system made up of a mathematical pendulum for which the pivot point (of negligible mass) is free to move horizontally. (10)

QUESTION B3 [*20 Marks*]

B3. (a) Using just definition of Hamiltonian $H(q,p)$ derive Hamilton's equations. (4)

(b) The Lagrangian for a certain system is given in Quesiton B1(d). Find

(i) Generalized momenta,

(ii) Hamiltonian,

(iii) Hamilton's equations. (3,5,3)

(c) Derive Hamilton's equations in Poisson's formulation. (5)

QUESTION B4 [*20 Marks*]

B4. (a) Derive Euler's equation corresponding to the functional

$$v[y(x)] = \int_{x_0}^{x_1} F(x, y(x), y'(x))dx, \quad y(x_0) = y_0, y(x_1) = y_1. \qquad (7)$$

(b) Derive an alternative form of Euler equation. (4)

(c) Let $F(y, y') = y\sqrt{1+(y')^2}$. Construct

(i) Euler's equation,

(ii) Beltrami's identity. (5,4)

QUESTION B5 [*20 Marks*]

B5. (a) Find the extremals of

(i) $v[y(x), z(x)] = \int_0^1 (y'^2 + z'^2 + y'z')dx$,

$y(0) = z(0) = 0, \quad y(1)$ is free, $z(1) = 2$.

(ii) $v[y(x)] = \int_0^1 (y''^2 + 1)dx$,

$y(0) = 0, y'(0) = y(1) = y'(1) = 1.$ [7,7]

(b) Write down Ostrogradski's equation for the following functionals

(i) $v[z(x,y)] = \int\int_D F(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y})dxdy$,

(ii) $v[z(x,y)] = \int\int_D \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right] dxdy.$ (3,3)

END OF EXAMINATION PAPER