UNIVERSITY OF SWAZILAND

FINAL EXAMINATION, MAY 2017

BA in Social Sciences I

Title of Paper : Elementary Quantitative Techniques II

Course Number : MAT102

Time Allowed : Two (2) Hours

Instructions

1. This paper consists of SEVEN (7) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 50%. Answer ALL questions in this section.
3. Section B consists of FOUR questions, each worth 25%. Answer ANY TWO (2) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B4 – B7) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A

Answer ALL Questions in this section

QUESTION A1

a . Evaluate

i. $\lim_{x\to 3}(2+4x^2-x^3)$ [2]

ii. $\lim_{x\to -2}\left(\frac{3x}{5-3x}\right)$ [2]

iii. $\lim_{x\to 4}\left(\frac{x^2-4x}{x-4}\right)$ [3]

iv. $\lim_{x\to \infty}\left(\frac{x+7}{x^2-2}\right)$ [4]

QUESTION A2

a. State the *limit definition* of the derivative of the function $f(x)$. [2]

b. Use the limit definition to find $\frac{df}{dx}$ given

$$f(x) = 5 - x^2$$

[7]

c. Find y' if

i. $y = 4 - 2x + \frac{2}{7}x^7$ [2]

ii. $y = 3X^{\frac{2}{3}} + X^{-\frac{4}{3}}$ [3]

iii. $y = 1 - \frac{2}{x^2}$ [3]

iv. $y = e^{2-5x}$ [2]

v. $y = \ln(3x)$ [3]

QUESTION A3

a. State the *Fundamental Theorem of Calculus.* [3]

b. Integrate

i. $\int_0^1 \left(3 + 4x - 15x^2\right) dx$ [5]

ii. $\int \left(7X^{\frac{1}{2}} + \frac{3}{X}\right) dX$ [3]

iii. $\int \left(\frac{5}{x^5} - \frac{3}{x^4}\right) dx$ [3]

iv. $\int e^{\frac{1}{2}x}\, dx$ [3]

Section B

Answer and 2 Questions in this section

QUESTION B4

a. Find the value of the limit of

$$\lim_{x \to -3} \frac{x+3}{x^2+4x+3}$$

[7]

b. Find the derivative

i. y' for $y = (1-4x^2)^{14}$ [3]

ii. y' for $y = (x+3)e^x$ [4]

iii. y' for $y = \frac{3x-2}{7x+4}$ [6]

iv. y''' for $y = 3\sqrt{x} + \frac{1}{x}$ [5]

QUESTION B5

a. Consider the function

$$y = 2 + 27x - x^3$$

i. Find the *stationary point(s)* and classify them as relative maxima or minima. [10]

ii. Find the y-intercept. [2]

iii. Make a neat sketch of the graph of y. [4]

b. The total costs C of a company (under certain conditions) are given by

$$C(x) = x^2 + \frac{16000}{x}$$

where x is the number of units produced per month. Find the number of units that minimises the monthly average costs. Hence find the minimum cost. [9]

QUESTION B6

a. By first making the substitution $u = x^2 + 1$, evaluate the integral

$$\int 6x\left(x^2+1\right)^5 dx.$$

[10]

b. Use the method of partial fractions to integrate

$$\int \frac{5x-6}{(x-1)(x-2)}\, dx.$$

[15]

QUESTION B7

a. Use the method of tabular integration to evaluate

$$\int 20x^2 e^x\, dx.$$

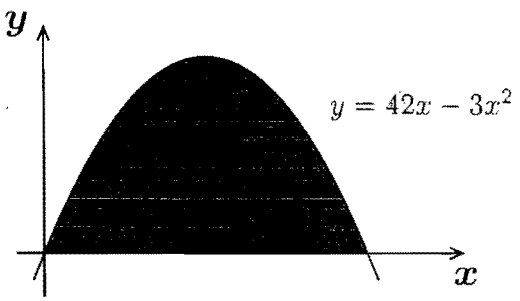
[12]

b. **i.** Evaluate

$$\int_0^1 \frac{2t^4 - 3t^3 + 5}{t^3}\, dt.$$

[7]

ii. Find the area of the shaded region in the figure below [6]



END OF EXAMINATION PAPER