

# University of Swaziland

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## Final Examination, May 2017

### B.A.S.S. I , B.Comm I, B. Ed, D.Comm I (IDE)

Title of Paper : Calculus for Business and Social Sciences

Course Code : MAT108/MAT122/MS102

Time Allowed : Three (3) Hours

#### Instructions

1. This paper consists of TWO sections.
  - a. **SECTION A (COMPULSORY): 40 MARKS**  
Answer ALL QUESTIONS.
  - b. **SECTION B: 60 MARKS**  
Answer ANY THREE questions.  
Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Special requirements: None

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

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QUESTION 1

(a) Evaluate the following limits

(i)  $\lim_{x \rightarrow -3} \frac{\sqrt{x^2 - 6x + 9}}{-x + 3}$  [2 marks]

(iii)  $\lim_{x \rightarrow -\infty} \frac{4 + 5x - 5x^2}{7 - 2x - 3x^2}$  [2 marks]

(ii)  $\lim_{x \rightarrow \frac{3}{2}} \frac{3x^3 + 27}{4x^2 - 9}$  [2 marks]

(b) Use the limit definition of the derivative to find the derivative  $f'(x)$  of the function

$$f(x) = \frac{1}{\sqrt{x}}$$

[4 marks]

(c) Given the function  $f(x) = \frac{4x^2 - 49}{2x - 7}$  [4 marks]

(i) Investigate the continuity of the function. (If the function is discontinuous then answer part (ii)).

(ii) Based on your answer from part (i) give a value of the function at the point of discontinuity which would make the function  $f(x)$  continuous.

(d) Find the derivatives of the following functions

(i)  $f(x) = (x^2 + 1) \sin^2 x$  [3 marks]

(ii)  $f(x) = \frac{\ln(2x + 5)}{x^2}$  [3 marks]

(e) Find the equation of the tangent line to the graph of

$$f(x) = x^3 - 4x$$

at  $x = 1$ .

[4 marks]

QUESTION 2

(a) Evaluate the following integrals

(i)  $\int_0^1 (3x - 2)^4 dx$  [3 marks]

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(ii)  $\int (x^3 + 3x)(x^2 + 1)dx.$  [3 marks]

(iii)  $\int (\cos 2x + e^{2x+3})dx.$  [3 marks]

(iv)  $\int x\sqrt{2x+1}dx.$  [3 marks]

(b) Find the area of the region bounded by

$$f(x) = x^2 \quad \text{and} \quad x = 9.$$

[4 marks]

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## SECTION B: ANSWER ANY 3 QUESTIONS

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### QUESTION 3

(a) Evaluate the following limits

(i)  $\lim_{x \rightarrow 3} \frac{3-x}{x^2-x-6}.$  [5 marks]

(ii)  $\lim_{x \rightarrow \infty} \sqrt{4x^2-2x+1} - 2x.$  [5 marks]

(iii)  $\lim_{x \rightarrow 2} \frac{4-x^2}{\sqrt{x}-\sqrt{2}}.$  [5 marks]

(b) Show that the function

$$f(x) = \begin{cases} 6x-2, & x \leq 1; \\ 4x^2, & x > 1 \end{cases}$$

is continuous at the point  $x = 1.$

[5 marks]

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### QUESTION 4

Find the derivative of the following functions

(a)  $f(x) = 3^{x^2-x+1}.$  [5 marks]

(b)  $f(x) = \left(\sqrt{x} + \frac{1}{x}\right)^5.$  [5 marks]

(c)  $f(x) = x^2 \tan 2x.$  [5 marks]

(d)  $f(x) = x^{\ln x}.$  [5 marks]

QUESTION 5

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- (a) Given the function

$$f(x) = x^4 - 2x^3,$$

find (if any) the

- (i) local maximum. [2 marks]
  - (ii) local minimum. [2 marks]
  - (iii) point of inflection. [2 marks]
- (b) Find the intervals where the curve is
- (i) increasing. [2 marks]
  - (ii) decreasing. [2 marks]
  - (iii) concave up. [3 marks]
  - (iv) concave down. [3 marks]
- (c) Use all the information obtained in (a) and (b) to sketch the graph of the function. [4 marks]

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QUESTION 6

- (a) A company's cost function is given by  $C(x) = \frac{x^2}{100} + \frac{x}{2} + 8$  and its price-demand function is given by  $p(x) = -\frac{x}{100} + 1$ .
- (i) Find the Revenue  $R(x)$  and Profit  $P(x)$  functions. [4 marks]
  - (ii) Find the marginal cost, marginal revenue and marginal profit functions. [4 marks]
  - (iii) Evaluate and interpret  $C(20), C'(20)$ . [4 marks]
- (b) A cattle owner has 800 meters of fence which he wishes to use to make a rectangular holding region in which his cattle will graze. If the region will border an existing fence, calculate the largest area that can be enclosed and find the dimensions of the region. [8 marks]

QUESTION 7

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(a) Evaluate the following integrals

(i)  $\int \frac{x}{x^2 + 5x + 6} dx.$

[5 marks]

(ii)  $\int \frac{\ln x}{\sqrt{x}} dx.$

[5 marks]

(b) Given the demand function  $D(x)$  and the supply function  $S(x)$

$$p = D(x) = 400 - 30x^2, \quad p = S(x) = 10x^2 + 120x$$

find the

(i) equilibrium price

[2 marks]

(ii) consumer surplus

[4 marks]

(iii) producer surplus

[4 marks]

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END