
University of Swaziland

33



Final Examination – May 2017

BSc I, BEd I, BEng I, BASS I

Title of Paper : Introduction to Calculus
Course Number : MAT112
Time Allowed : Three (3) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY THREE (3) questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

34

Section A
Answer ALL Questions in this section

A.1 a. $\lim_{x \rightarrow \frac{1}{2}} \left(\frac{2x^2 + x - 1}{2x^2 - 5x + 2} \right)$ [4 marks]

b. $\lim_{\theta \rightarrow 0} \left(\frac{6\theta + \sin 4\theta}{\theta} \right)$ [3 marks]

A.2 Find $\frac{dy}{dx}$ if

a. $y = (1 + 4\sqrt{x})^{20}$ [3 marks]

b. $y = \sin^{-1}(\cos x)$ [4 marks]

c. $x^2y - y^4 = 5$, where $y = y(x)$ [4 marks]

A.3 Use the *limit definition* to find $\frac{df}{dx}$ given

$f(x) = 7 - 3x^2$. [7 marks]

A.4 Integrate

a. $\int_0^1 4xe^{-2x^2} dx$ [3 marks]

b. $\int_0^1 4x^2e^{-2x} dx$ [4 marks]

c. $\int_0^{\frac{\pi}{4}} 8 \cos^3 \theta \sin \theta d\theta$ [4 marks]

d. $\int \frac{dx}{1-x^2}$ [4 marks]

Section B

Answer ANY THREE (3) Questions in this section

35

B.1 a. Make a sketch of the graph of

$$y = 2H(x + 2) + H(x - 2),$$

where $H(x)$ is the Heaviside function of x . [4 marks]

b. Evaluate

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2 - 8x})$$
 [6 marks]

c. Use the *limit definition* to find y' given

$$y = 7 + \frac{8}{\sqrt{x}}.$$
 [10 marks]

B.2 a. Find $\frac{dy}{dx}$ and simplify given

$$y = \tan^{-1}\left(\frac{x}{3}\right) + \frac{3x}{x^2 + 9}.$$
 [6 marks]

b. Consider the function

$$y = 10 + x^2 - 10 \ln(1 + x^2).$$

i. Find the stationary points of y . [6 marks]

ii. Determine the nature of each stationary point. [4 marks]

iii. Make a sketch of the graph of y . [4 marks]

B.3 a. Differentiate the function

$$y = \frac{\sin \theta}{\sin \theta - \cos \theta}$$

and show that

$$\frac{dy}{d\theta} = \frac{1}{\sin 2\theta - 1}.$$
 [8 marks]

b. Find the dimensions of the *largest* rectangle that can be inscribed inside the ellipse

$$\frac{x^2}{36} + \frac{y^2}{9} = 1.$$
 [12 marks]

B.4 a. Integrate

i. $\int (4x + 1) \ln x \, dx$ [4 marks]

ii. $\int_0^4 \frac{x^2 \, dx}{16 + x^2}$ [7 marks]

b. Derive the formula

$$A = \pi ab$$

for the area of an ellipse with major axis a and minor axis b . [9 marks]

B.5 Integrate

a. $\int \frac{x^4 - x^3 + x^2}{x^3 - x^2 - x + 1} \, dx$ [15 marks]

b. $\int \frac{d\theta}{1 + e^\theta}$ [5 marks]

END OF EXAMINATION
