UNIVERSITY OF SWAZILAND

FINAL EXAMINATION, 2016/2017

B.Sc. II

Title of Paper : Mathematics for Scientists

Course Number : MAT215/M215

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.

   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three [redacted]wered in Section B will be marked.**
2. Show all your working.

Special Requirements: None

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: Answer ALL Questions

64

A1. (a) Use a scalar product to find a unit vector perpendicular to both $\bar{a} = (2, 3, -1)$ and $\bar{b} = (3, -2, 1)$. (5)

A2. State Rolle's Theorem. (3)

A3. Compute $\lim_{x \to 0^+} \frac{\sin x}{\sqrt{x}}$. (3)

A4. State Taylor's Series Theorem. (3)

A5. If $f(x, y) = x \exp(2x^2 + y^2)$, find f_x and f_y. (3)

A6. State the equality of Mixed Derivatives Theorem. (2)

A7. For the function $f(x, y, z) = xy + yz - xz + x$, find

(a) Gradient,

(b) stationary points. (2,2)

A8. Minimize $f(x, y) = x^2 + y^2$, subject to constraint $2x + y = 2$. (5)

A9. Compute the volume under the graph of $z = 2x + y$ over the region $2 < x < 4$, $1 < y < 6$. (5)

A10. Use polar coordinates to compute $\int\int_D \sqrt{x^2 + y^2} \quad dxdy$ if D is region in the first quadrant bounded by the unit circle and the coordinate axes.. (5)

A11. Define an ODE with homogeneous coefficients. (2)

SECTION B: Answer Any *THREE* Questions

QUESTION B1 [*20 Marks*]

B1. (a) Does point $R(1, 3)$ lie on the line through $P(-1, -2)$ and $Q(\frac{1}{2}, \frac{7}{4})$? (5)

(b) Use the vector product to find a vector perpendicular to $\bar{a} = (1, 0, 0)$ and $\bar{b} = (1, 0, \sqrt{3})$. (5)

(c) Find the volume of parallelepiped spanned by the directed segments $\overline{OA}, \overline{OB}$ and $\overline{OC}$ if the coordinates of A, B and C are $(8, 4, 0)$, $(2, 6, 0)$ and $(0, 4, 6)$ respectively. (5)

(d) If $f(x) = \frac{4}{x}$, find (if possible) all numbers in the interval $(1, 4)$ for which the Mean Value Theorem is satisfied. Explain. (5)

65

QUESTION B2 [*20 Marks*]

B2. (a) Evaluate (i) $\lim_{x\to+\infty} \frac{\ln x}{x}$,

(ii) $\lim_{x\to+\infty} \frac{x \ln x}{(x+1)^2}$ (4,4)

(b) (i) Use the quadratic approximation formula to compute e^x for small $|x|$, and estimate the error.

(ii) In particular compute $e^{-0.1}$. (4,3)

(c) Find the third Taylor's polynomial at $x_o = \frac{\pi}{6}$ for $f(x) = \sin x$. (5)

QUESTION B3 [*20 Marks*]

B3. (a) Use the chain rule to evaluate f_u and f_v if $f(x,y) = 3x^2y$ and

$x = u + v, \quad y = uv.$ (6)

(b) Verify equality of Mixed Derivatives Theorem for $f(x,y) = \sqrt{2x - y}$. (4)

(c) If $f(x,y) = \ln\left(\frac{x+1}{y+1}\right)$, what is df? (5)

(d) Find and classify all stationary points of $f(x,y) = x^4 + y^4 + 4x - 4y$. (5)

QUESTION B4 [*20 Marks*]

B4. (a) A farmer who wants to fence a rectangular grazing field bordering a straight river has 600m of fencing material. If the side along the stream will not be fenced, what length and width will provide the maximum grazing area? Apply Lagrange method. (6)

(b) Let R be the triangular region bounded by the lines $2y = x, y = 2x$ and $x = \pi$. compute $\int\int_R \sin y dxdy$. (6)

(c) Compute $\int\int\int_\Delta \frac{x+y}{z} dxdydz$, where

$\Delta = \{(x,y,z): \ -1 < x < 1, \ \ -2 < y < 2, \ \ 1 < z < 2\}$ (8)

QUESTION B5 [*20 Marks*]

B5. (a) Solve $y' = \frac{2y}{x}$ for $x > 0, y > 0$. (5)

(b) Prove that if $f(x,y)$ is homogeneous of degree zero, then $f(x,y)$ is function of $\frac{y}{x}$ alone. (5)

(c) Test $tdt + xdx = 0$ for exactness and solve it. (4)

(d) Solve $y'' - 6y' + 9y = 0, \quad y(0) = 0, \quad y'(0) = 1.$ (6)

END OF EXAMINATION PAPER