University of Swaziland

Supplementary Examination, 2016/2017

B.Sc. II, B.Eng II, BASS II, BED. II

Title of Paper : Linear Algebra

Course Number : M220

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.

   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked.**
2. Show all your working.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

A1.

(a) Find conditions on λ and u for which the following system of linear equations has

(i) a unique solution

(ii) no solution

(iii) infinitely many solutions

$$\begin{aligned} x+y+z &= 0 \\ 2x+3y+z &= 1 \\ 4x+7y+\lambda z &= u \end{aligned} \qquad (1)$$

(10)

(b) Let $T : R^2 \to R^3$ be given by

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x-2y \\ 2x+y \\ x+y \end{pmatrix}$$

Find the matrix of T

(i) with respect to the standard basis

(ii) with respect to B^1 and B where

$B^1 = \{(1,-1)^T, (0,1)^T\}$ and $B = \{(1,1,0)^T, (0,1,1)^T, (1,-1,1)^T\}$ (10)

A2.

(a) Determine whether the following sets of vectors in the vector space $P_2(x)$ are linearly dependent. For those that are linearly dependent express the last vector as a linear combination of the rest

(i) $\{2x^2+x, x^2+3, x\}$

(ii) $\{2x^2+x+1, 3x^2+x-5, x+13\}$ (10)

(b) Let $S = \{\gamma_1, \gamma_2 \cdots, \gamma_n\}$ be a set of non-zero vectors in a vector space V. Prove that S is linearly dependent if and only if one of the vectors γ_j is linear combination of the proceeding vectors in S. (10)

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B3 [*20 Marks*]

(a) Let V be the set of all ordered pairs of real numbers. Define addition and scalar multiplication as follows

$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2 + 1, y_1 + y_2 + 1)$ and $\alpha(x, y) = (\alpha x_1 + \alpha - 1, \alpha y_1 + \alpha - 1)$

Show that V is a vector space (8)

(b) Find the inverse A^{-1} of the matrix A using the augmented matrix $[A : I]$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & -1 & 2 \end{pmatrix}$$ (10)

(c) Use (b) to find a finite sequence of elementary matrices $E_1, E_2, \cdots, E_k$ such that $E_k E_{k-1} \cdots E_2 E_1 A = 1$ (2)

QUESTION B4 [*20 Marks*]

(a) Show that the vector $(-3, 12, 12)$ is a linear combination of the vectors $(1, 0, 2)$, $(0, 2, 4)$ and $(-1, 3, 2)$ (8)

(b) Show that the set of vectors

$V = \{(0, 2, 1)^T, (1, 0, 2)^T, (1, -1, 0)^T\}$ is basis for R^3 (8)

(c) Determine whether the following has a non trivial solution

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ 2x_1 + x_2 - x_3 + 2x_4 &= 0 \\ 3x_1 + 2x_2 + 2x_3 + 2x_4 &= 0 \end{aligned}$$

(4)

QUESTION B5 [*20 Marks*]

(a) Show that each eigen vector of a square matrix A is associated with only one eigen-value (5)

(b) Show that $A = \begin{pmatrix} 0 & 0 & 5 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$

is a skew symmetrix (5)

(c) Find the characteristic polynomial, eigen values and eigen vectors of the following matrix

$A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{pmatrix}$ (10)

QUESTION B6 [*20 Marks*]

Let $B = \{u_1, u_2, u_3\}$ and $B^1 = \{\gamma_1, \gamma_2, \gamma_3\}$ be bases in R^3, where

$u_1 = (1,0,0)^T, u_2 = (1,1,0)^T, u_3 = (1,1,1)^T$

$\gamma_1 = (0,2,1)^T, \gamma_2(1,0,2)^T, \gamma_3 = (1,-1,0)^T$

(a) Find the transition matrix from B^1 to B. (10)

(b) Let $T : R^3 \to R^3$ be the linear transformation whose matrix with respect to the basis B is

$$\begin{pmatrix} 3 & -6 & 9 \\ 0 & 3 & -6 \\ 0 & 0 & 0 \end{pmatrix}$$

Find the matrix of T w.r.t B^1 (2)

(c) If $\begin{pmatrix} 6 \\ -3 \\ 3 \end{pmatrix}$ is the coordinates relative to B, find the coordinates relative to B^1 (6)

QUESTION B7 [*20 Marks*]

(a) Verify Cayley-Hamilton Theorem for the following matrix

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$

(5)

(b) By inspection, find the inverses of the following matrices

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(5)

(c) Use Crammer's rule to solve

$$\begin{aligned} 2x_1 + 8x_2 + x_3 &= 10 \\ 2x_3 + 3x_2 - x_1 &= -2 \\ 4x_1 + 4x_2 - 5x_3 &= 4 \end{aligned}$$

(5)

(c) Show that $2x^2 + 2x + 3$ is not a linear combination of
$x^2 + 2x + 1, \quad x^2 + 3, \quad x - 1$ (5)

END OF EXAMINATION PAPER