University of Swaziland

Final Examination, December 2017

B.A.S.S. , B.Sc, B.Ed

Title of Paper : Dynamics II

Course Number : M355

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: Answer All Questions

A1. Derive the Lagrangian for the masses of Atwood machine, taking the shorter length of the pulley to be the generalized coordinate. [8]

A2. The Lagrangian function of a system is given by

$$\frac{1}{2}M\dot{R}^2 + \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{1}{2}k(r-b)^2.$$

(a) Determine the cyclic (ignorable) coordinates and find the generalized momenta conjugate to these coordinates. [4]

(b) Prove that the Hamiltonian of the system is given by

$$\frac{P_R^2}{2M} + \frac{P_r^2}{2\mu} + \frac{P_\theta^2}{2\mu r^2} + \frac{1}{2}k(r-b)^2.$$

[5]

(c) Determine Hamilton's equations of motion and prove that the equation of motion corresponding to r is

$$\mu(\ddot{r} - r\dot{\theta}^2) + k(r-b) = 0.$$

[5]

A3. (a) Evaluate the poison bracket $[q^2p, qp]$. [6]

(b) Using any method you choose, show that the following transformation is canonical

$$P = \frac{1}{2}(p^2 + q^2), \quad Q = \arctan\left(\frac{q}{p}\right).$$

[6]

A4. Find the extremal of

$$I = \int_0^1 ((y'')^2 + y' + 3x^2)dx, \quad y(0) = 0, \quad y(1) = 1, \quad y'(0) = 1, \quad y'(1) = 1$$

[6]

SECTION B: Answer Any THREE Questions

B2. (a) Derive the Lagrange equation of motion for the system made up of a simple pendulum for which the pivot point (of negligible mass) is free to move horizontally as shown in Figure 1: Take the horizontal line through $y = 0$ as the reference level.

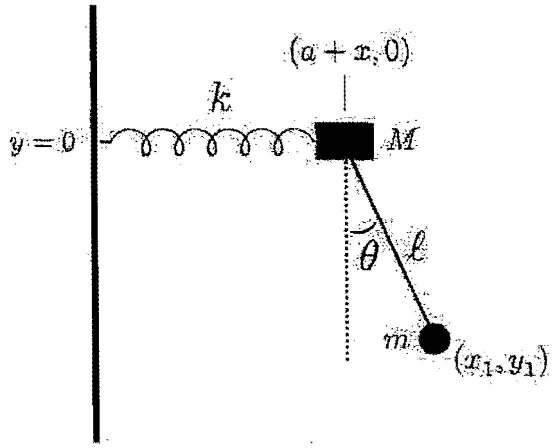

Figure 1: Moving Pivot Pendulum

[12]

(b) For a certain mechanical system the kinetic and potential energy are given by

$$\begin{aligned} T &= \frac{1}{2}\{(1+2k)\dot{\theta}^2 + 2\dot{\theta}\dot{\phi} + \dot{\phi}^2\}, \\ V &= \frac{n^2}{2}\{(1+k)\theta^2 + \phi^2\}, \end{aligned}$$

where θ and ϕ are generalized coordinates and n, k are positive constants. Write down Lagrange's equations of motion and deduce that

$$(\ddot{\theta} - \ddot{\phi}) + n^2\left(\frac{1+k}{k}\right)(\theta - \phi) = 0.$$

[8]

B3. Consider a particle moving on a real line. Let the dynamics of the particle be determined by the Hamiltonian,

$$H = \frac{q^4 p^2}{2\mu} + \frac{\lambda}{q^2},$$

where μ and λ are real constants.

(a) Write down the Hamilton's equations of motion for the above system in their simplest form. [8]

(b) Find a Lagrangian for the system and write down the corresponding Lagrange's equation of motion. [12]

B4. (a) Using any method you choose, prove that the following transformation is canonical

$$Q = q \tan p, \quad P = \ln(\sin p)$$

[8]

(b) Prove that $\frac{d}{dt}[A, B] = [\dot{A}, B] + [A, \dot{B}]$. [6]

(c) Evaluate $[A_1, A_2]$ and $[A_1, A_3]$ given

$$A_1 = \frac{1}{4}(x^2 + p_x^2 - y^2 - p_y^2), \quad A_2 = \frac{1}{2}(xy + p_x p_y), \quad A_3 = \frac{1}{2}(xp_y - yp_x).$$

[6]

B5. (a) Find the curve $y(x)$ that minimizes the functional

$$\int_0^1 (y'^2 + y^2 + 2ye^{2x})dx, \quad y(0) = \frac{1}{3}, \quad y(1) = \frac{1}{3}e^2.$$

[10]

(b) Show that Euler-Lagrange equation for the functional

$$I = \int_{x=a}^{b} y\sqrt{1+(y')^2}dx,$$

is given by

$$yy'' = 1 + (y')^2.$$

[10]

B6. (a) A system of two degrees of freedom is described by the Hamiltonian

$$H = q_1p_1 - q_2p_2 - aq_1^2 + bq_2^2,$$

where a and b are constants. Show that $D = q_1q_2$, is a constant of motion. [6]

(b) Find the function which optimises

$$\int_{x=1}^{e} (4y + xy'^2)dx$$

if $y(e) = 0$ and $y(1)$ not prescribed. [14]

END OF EXAMINATION