University of Swaziland

Supplementary Examination, July 2018

B.A.S.S. , B.Sc, B.Ed

Title of Paper : Dynamics II

Course Number : M355

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: Answer All Questions

A1. (a) Define generalized coordinates, giving examples. [3]

(b) A particular mechanical system depending on two coordinates u and ν has kinetic energy $T = \nu^2\dot{u}^2 + 2\dot{\nu}^2$, and potential energy $V = u^2 - \nu^2$. Write down the Lagrangian for the system and deduce the system's equations of motion. [6]

A2. (a) Define the Hamiltonian function H in terms of the Lagrangian function, L. [3]

(b) Derive the Hamilton's equations when H does not depend on time t, explicitly. [6]

A3. Show that the following transformation of one degree of freedom is canonical

$$P = \frac{1}{2}(p^2 + q^2), \quad Q = \tan^{-1}\left(\frac{q}{p}\right).$$

[8]

A4. (a) Define a Poisson bracket between two physical quantities $F(q_\alpha, p_\alpha, t)$ and $G(q_\alpha, p_\alpha, t)$. [3]

(b) Consider a system with Hamiltonian $\frac{1}{2}\left(p_1^2 + q_1^2 + p_2^2 + q_2^2\right)$. Show that $M = p_1 p_2 + q_1 q_2$ is a constant of motion by evaluating the Poisson bracket $[M, H]$. [6]

A5. Find the curve $y(x)$ that minimizes the functional

$$\int_0^{\frac{\pi}{2}} (y'^2 - y^2)dx, \quad y(0) = 0, \quad y\left(\frac{\pi}{2}\right) = 1.$$

[5]

SECTION B: Answer Any THREE Questions

B2. (a) A double pendulum consists of two simple pendulums of lengths l_1 and l_2 and masses m_1 and m_2, with the cord of one pendulum attached to the bob of another pendulum whose cord is fixed to a pivot, (see Figure 1). Determine the equations of motion for small angle oscillations using Lagranges equations. [12]

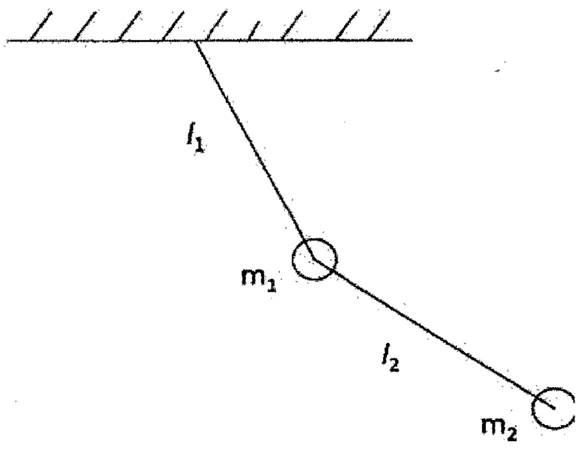

Figure 1: Double Pendulum

(b) A particle moves in the xy-plane subject to the Lagrangian

$$\frac{1}{2}(\dot{x}^2 + \dot{y}^2) + \Omega(-\dot{x}y + \dot{y}x),$$

where Ω is a constant. Find the Hamiltonian of the system and show that it is conserved. [8]

B3. Suppose that a system has a Hamiltonian defined by

$$\frac{1}{2m}(p_x^2 + m^2\omega^2 x^2) + \frac{1}{2m}(p_y^2 + m^2\omega^2 y^2).$$

(a) Show, using Hamilton's equations, that the equations of motion can be written as

$$\ddot{x} + \omega^2 x = 0, \quad \ddot{y} + \omega^2 y = 0.$$

[10]

(b) Evaluate the Poisson bracket $[G, H]$ where

$$G = \frac{1}{2m\omega}(p_x p_y + m^2\omega^2 xy), \quad H = \frac{1}{4m\omega}(p_x^2 - p_y^2 + m^2\omega^2(x^2 - y^2)).$$

[10]

B4. (a) Prove that for a system whose dynamic behavior defined is by the Hamiltonian $H = H(q_\alpha, p_\alpha, t)$, the equation of motion for a dynamic variable $f(q_\alpha, p_\alpha, t)$ is

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [f,\ H].$$

[4]

(b) The Hamiltonian of a two-dimensional harmonic oscillator of unit mass is given by

$$H = \frac{1}{2}(p_1^2 + p_2^2) + \frac{1}{2}\omega(q_1^2 + q_2^2)$$

where ω is a constant. Given that $f = \omega q_1 \sin \omega t + \omega p_1 \cos \omega t$, show that f is a constant of motion. [8]

(c) For what values of the constant parameters α and β is the transformation below canonical?

$$q = \beta P^\alpha \sin Q, \quad p = \beta P^\alpha \cos Q.$$

[8]

B5. (a) A system of two degrees of freedom is described by the Hamiltonian

$$H = q_1 p_1 - q_2 p_2 - a q_1^2 + b q_2^2,$$

where a and b are constants. Show that

$$F_1 = \frac{p_1 - a q_1}{q_2}, \quad \text{and } F_2 = q_1 q_2,$$

are constants of motion. [10]

(b) Find the curve $y(x)$ that minimises the following functional

$$\int_0^1 (y' + (1+x)y'^2)dx, \quad y(0) = 0, \quad y(1) = \ln 2.$$

[10]

B6. (a) Show that if F has no explicit dependence on x (i.e $\partial F/\partial x = 0$) then $F = F(y, y')$ and the Euler-Lagrange's equation simplifies to the Beltrami-identity which is defined by the equation

$$F - y'\frac{\partial F}{\partial y'} = C$$

where C is a constant. [10]

(b) Use the Beltrami identity to show that the extremum for the integral

$$I = \int_{x=0}^{a} \sqrt{\frac{1 + y'^2}{2y}} dx$$

is given by

$$y' = \sqrt{\frac{c - y}{y}}.$$

[10]

END OF EXAMINATION