University of Swaziland

Main Examination, 2017/2018

B.Sc. IV, BASS IV, B.Ed IV

Title of Paper : Fluid Dynamics

Course Number : M455

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section. If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked**
4. Show all your working.
5. Start each new major question (A1, B1-B5) on a new page and clearly indicate the question number at the top of the page.
6. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

A1. Among four states of matter define

(a) Solids, [2]

(b) Liquids. [2]

A2. Describe the Lagrange method of treating motion of continuum medium. [4]

A3. A velocity field in polar is given as

$\overline{V} = \frac{A}{r}(\overline{e}_r + \overline{e}_\theta)$

Find the stream line passing through the point $(r, \theta) = (1, 0)$. [4]

A4. Does the following set of velocity components $u = 2xy - x^2 + y$, $v = 2xy - y^2 + x^2$, represent two-dimensional

(a) incompressible flow, [3]

(b) irrotational flow? [3]

A5. Explain the term τ_{xz}. [3]

A6. A closed container contains water at a 5m depth. The absolute pressure above the water surface is 0.3 atm. Calculate the absolute pressure on the inside of the bottom surface of the container.

Hint: 1 atm = 101.3 kpa. [5]

A7. Write down

(a) Euler equation of the motion, [3]

(b) Novier-Stoke's equation (NSE) for incopressible flow. [3]

(c) NSE in dimensionless form. [3]

A8. Apply Euler equation for potential force field and vector identity given in "useful formulae" to show that

$$\overline{V} \times \overline{\omega} = \nabla \left(\frac{1}{2}V^2 + \phi + \frac{p}{\rho} \right)$$

in the usual notations. [5]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B1 [*20 Marks*]

a) Define a density at a point using a continuum model. [3]

b) Consider the flow field given in Eulerian description by the expression $\overline{V} = Ai + Btj$, where $A = 2\frac{m}{s}, B = 0.3\frac{m}{s^2}$, and the coordinates are measured in meters. Derive the Lagrange position functions for the field particle that was located at the point $(x, y) = (1, 1)$ at the instant $t = 0$.

i) Obtain an algebraic expression for the pathline followed by this particle. [4]

ii) Plot the pathline and compare with the streamlines through the same point at the instants $t = 0, 1$ and 2 s [4]

c) Derive the formula for convective derivative of the density. [6]

d) For the flow in xy plane, the y component of velocity is given by

$$v = y^2 - 2x + 2y.$$

Determine a possible x component for steady, incompressible flow. [3]

QUESTION B2 [*20 Marks*]

a) Prove $\overline{V} = \nabla\psi \times \overline{k}$ in the usual notations. [3]

b) Consider the flow field given by

$$\psi = 9 + 6x - 4y + 7xy.$$

i) Show that flow is irrotational. [3]

ii) Determine the velocity potential ϕ for this flow. [3]

iii) Show that lines of constant ψ and ϕ are orthogonal. [3]

(c) The stream function for a certain incompressible flow field is given by the expression

$$\psi(r, \theta) = -Ur\sin\theta + \frac{q\theta}{2\pi}.$$

i) Obtain an expression for the velocity field. [3]

ii) Find the stagnation point, and [3]

iii) Show that $\psi = 0$ there. [2]

QUESTION B3 [*20 Marks*]

a) Prove that the pressure at a point of fluid is the same in all directions. [6]

b) Derive relation of pressure to body forces. [4]

c) State and prove Archimedes' theorem. [6]

d) Define the Newtonian fluid. [4]

QUESTION B4 [*20 Marks*]

a) The velocity distribution in two-dimensions flow is given by

$$\overline{V} = (Ax + By)i - Ayj, \quad \text{where} \quad A = 1s^{-1}, B = 2s^{-1},$$

and the coordinates are measured in meters.

i) Is it a possible incompressible flow? [2]

ii) Find the acceleration of fluid particle at point $(x, y) = (1, 2)$. [5]

iii) Find the pressure gradient at the same point, if $\overline{g} = -g\overline{j}$ and the fluid is water. [5]

iv) Find the pressure distribution along the x-axis if $p(0, 0) = 100pa$. [3]

b) A tank partly filled with water is subject to a constant linear horizontal acceleration a_x. Using the Euler equation, find the shape of the free surface. [5]

QUESTION B5 [*20 Marks*]

a) Consider steady, incompressible viscous flow in a cylindrical pipe of radius a (Poiseulle flow). Neglect body forces. Pressure gradient is G. Pressure at $z = 0$ is P_0. Put $\overline{V} = u(r)\overline{k}$ and apply Navier-Stoke's equations to show that

i) $P = P(z) = P_0 - Gz$, [5]

ii) $u(r) = \frac{G}{4\mu}(a^2 - r^2)$. [5]

b) Air flows steadily through a horizontal nozzle, discharging to the atmosphere. At the nozzle inlet, the area is $0.1m^2$. At the nozzle exit, the area is $0.02m^2$. The flow is incompressible, and frictional effects are negligible. Air density is $1.23kg/m^3$. Apply Bernoulli's equation to determine the gage pressure required at the nozzle inlet to produce an outlet speed of $50m/s$. [10]

END OF EXAMINATION PAPER

USEFUL FORMULAE

The gradient of a function $\psi(r, \theta, z)$ in cylindrical coordinates is

$$\nabla\psi = \frac{\partial \psi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial \phi}{\partial \theta}\hat{\theta} + \frac{\partial \psi}{\partial z}\hat{k}$$

The divergence and curl of the vector field

$$\underline{v} = v_r\hat{r} + v_\theta\hat{\theta} + v_z\hat{k}$$

in cylindrical coordinates are

$$\nabla \cdot \underline{v} = \frac{1}{r}\left\{\frac{\partial}{\partial r}(rv_r) + \frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(rv_z)\right\}$$

and

$$\nabla \times \underline{v} = \frac{1}{r}\det\begin{bmatrix} \hat{r} & r\hat{\theta} & \hat{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ v_r & rv_\theta & v_z \end{bmatrix}$$

The divergence of a vector

$$\underline{v} = v_r\hat{r} + v_\lambda\hat{\lambda} + v_\theta\hat{\theta}$$

in spherical coordinates

$$\nabla \cdot \underline{v} = \frac{1}{r^2}\frac{\partial (r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial v_\lambda}{\partial \lambda} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, v_\theta)}{\partial \theta}$$

The convective derivative and Laplacian in cylindrical coordinates are

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial \theta} + v_z\frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

Identities

$$\underline{v} \cdot \nabla\underline{v} = \nabla\left(\frac{v^2}{2}\right) - \underline{v} \times \underline{\omega}$$

$$\nabla \times (\nabla \times \underline{a}) = \nabla\nabla \cdot \underline{a} - \nabla^2\underline{a}$$