UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION, 2017/2018

B.Sc. IV, BASS IV, B.Ed IV

Title of Paper : Fluid Dynamics

Course Number : M455

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.
   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked.**
2. Show all your working.

Special Requirements: -None

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: Answer ALL Questions

A1. Give essential features of

(a) liquids,

(b) gases. (2,2)

A2. Describe the Euler method of treating motion of continuum medium. (4)

A3. A tornado can be represented in polar coordinates by the velocity field

$$\overline{V} = -\frac{a}{r}\overline{e}_r + \frac{b}{r}\overline{e_\theta}.$$

Find the stream lines. (4)

A4. Does the following set of velocity components $u = xt + 2y, v = xt^2 - yt$, represent possible two-dimensional

(a) incompressible flow,

(b) irrotational flow? (3,3)

A5. Explain the term τ_{zy}. (3)

A6. On the water surface the pressure is 1 atm (101.3kpa). How deep one can dive to get a maximum pressure of 2 atm? (5)

A7. (a) Define Reynolds number,

(b) Define similar flows.

(c) How the notion of the similar flows is used in experiments? (3,3,3)

A8. Consider steady, incompressible, inviscid, potential flow. Given

$$\overline{V} \times \overline{\omega} = \nabla(\frac{V^2}{2} + \phi + \frac{p}{\rho}),$$

in the usual notations. Derive Bernoullis equation. (5)

SECTION B: Answer any *THREE* Questions

QUESTION B1 [*20 Marks*]

B1. (a) Define a continuum model on example of air density (3)

(b) Parametric equations for the position of a particle in a flow field are given as $x_p = c_1e^{at}$ and $y_p = c_2e^{-bt}$.

(i) Find the equation of the pathline for a particle located at $(x, y) = (1, 2)$ at $t = 0$.

(ii) Show that for this flow field $\overline{V} = axi - byj$.

(iii) Compare the pathline with streamline through the same point. (3,2,3)

(c) Derive the formula for mass conservation (continuity equation). (6)

(d) The x component of velocity in steady, incompressible flow field in the xy plane is $u = \frac{A}{x}$. Find the simplest y component of velocity for this flow field. (3)

QUESTION B2 [*20 Marks*]

B2. (a) Prove $\overline{V} = \nabla \times \psi\overline{k}$ in the usual notations (3)

(b) Consider the flow field given by $\psi = -2xy + x + 3$.

(i) Show that the flow is irrotational.

(ii) Determine the velocity potential for this flow.

(iii) Show that lines of constant ψ and ϕ are orthogonal. (3,3,3)

(c) Incompressible flow around a circular cylinder of radius a is represented by the stream function

$$\psi(r, \theta) = -Ur\sin\theta + \frac{Ua^2\sin\theta}{r},$$

where U represents the free stream velocity (or velocity of infinity).

(i) Obtain an expression for the velocity field.

(ii) Find v_r along the circle $r = a$.

(iii) Locate the points along $r = a$, where $|\overline{V}| = U$ (3,2,3)

QUESTION B3 [*20 Marks*]

B3. (a) Consider a fluid at rest in the field of gravity. Derive equilibrium equations. (6)

(b) Consider liquid uniformly rotating with angular velocity ω in the field of gravity.

(i) Construct equilibrium equations.

(ii) Find the equation of the upper surface of the liquid. (4,4)

(c) State and prove Archimede's theorem. (6)

QUESTION B4 [*20 Marks*]

B4. (a) A velocity field in a fluid with density of $1500 kg/m^3$ is given by

$$\overline{V} = (Ax - By)t\overline{i} - (Ay + Bx)t\overline{j}$$

where $A = 1s^{-2}$, $B = 2s^{-2}$, and x and y are in meters and t is seconds. Body and viscous forces are negligible.

(i) Is it a possible incompressible flow?

(ii) Find the acceleration of a fluid particle at point $(x, y) = (1, 2)$ at $t = 1sec$.

(iii) Find the pressure gradient at the same point and same time.

(iv) Find the pressure distribution along the x-axis if $p(0,0) = p_0$. (2,6,5,3)

(b) (i) Write constitutive (essential) relations for Newtonian fluid.

(ii) Define a kinematic viscosity. (2,2)

QUESTION B5 [*20 Marks*]

B5. (a) Consider a cylinder of radius a rotating with angular velocity ω in a viscous incompressible fluid. Put $\overline{V} = u(r)\overline{e_\theta}$ and apply Novier-Stoke's equations to show that

$$u = \begin{cases} \omega r & if \quad r \leq a \\ \frac{\omega a^2}{r} & if \quad r \geq a. \end{cases}$$

(10)

(b) Water flows in circular pipe. At one section the diameter is $0.3m$, the static pressure is $260kpa$ (gage), the velocity is $3m/s$, and the elevation is 10m above ground level. At a section downstream at ground level, the pipe diameter is 0.15m. Find the gage pressure at the downstream section, if friction effects may be neglected. (10)

END OF EXAMINATION PAPER

USEFUL FORMULAE

The gradient of a function $\psi(r, \theta, z)$ in cylindrical coordinates is

$$\nabla\psi = \frac{\partial\psi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{\theta} + \frac{\partial\psi}{\partial z}\hat{k}$$

The divergence and curl of the vector field

$$\underline{v} = v_r\hat{r} + v_\theta\hat{\theta} + v_z\hat{k}$$

in cylindrical coordinates are

$$\nabla \cdot \underline{v} = \frac{1}{r}\left\{\frac{\partial}{\partial r}(rv_r) + \frac{\partial}{\partial\theta}(v_\theta) + \frac{\partial}{\partial z}(rv_z)\right\}$$

and

$$\nabla \times \underline{v} = \frac{1}{r}\det\begin{bmatrix} \hat{r} & r\hat{\theta} & \hat{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial z} \\ v_r & rv_\theta & v_z \end{bmatrix}$$

The divergence of a vector

$$\underline{v} = v_r\hat{r} + v_\lambda\hat{\lambda} + v_\theta\hat{\theta}$$

in spherical coordinates

$$\nabla \cdot \underline{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial v_\lambda}{\partial\lambda} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, v_\theta)}{\partial\theta}$$

The convective derivative and Laplacian in cylindrical coordinates are

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial\theta} + v_z\frac{\partial}{\partial z}$$
$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}$$

Identities

$$\underline{v} \cdot \nabla\underline{v} = \nabla\left(\frac{v^2}{2}\right) - \underline{v} \times \underline{\omega}$$
$$\nabla \times (\nabla \times \underline{a}) = \nabla\nabla \cdot \underline{a} - \nabla^2\underline{a}$$