# University of Swaziland

## Final Examination, December 2017

## B.A.S.S I

**Title of Paper** : Elementary Quantitative Techniques I

**Course Number** : MAT101

**Time Allowed** : Three (3) Hours

**Instructions**

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      **Submit solutions to ONLY THREE questions in Section B.**
2. Show all your working.
3. Start each question on a fresh page.
4. Non programmable calculators may be used (unless otherwise stated).
5. Leave all non exact answers correct to 3 decimal places unless told otherwise.
6. A formula sheet is provided on the last page.
7. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

# SECTION A

Answer ALL questions from Section A.

A1. (a) Simplify:

i. $(a^3b^3)(a^{-2}b^3)$. [2]

ii. $\dfrac{b^2-4b}{b^2-10b+24}$. [3]

iii. $\dfrac{1}{x+3}+\dfrac{1}{x-2}$. [3]

iv. $\log x+\log(x+5)$ [2]

(b) Evaluate

i. $\log_b b^3$. [2]

ii. $\log_2 16$. [2]

iii. $16C_7$. [2]

iv. $\ln 457$. [2]

(c) Factorise completely $4n^2+12n+9$. [4]

(d) Evaluate $\begin{vmatrix} 4 & -4 & 2 \\ 0 & 9 & 3 \\ 5 & 0 & 3 \end{vmatrix}$. [5]

(e) Solve

$$3m+2n=4,$$
$$2m+3n=11.$$

[5]

(f) Find the equation of the line passing through $(4,0)$ and parallel to the line $y=3x-5$. [3]

(g) For the progression $10, 2, -6, \ldots$, find

i. the formula for the $n$th term. [3]

ii. $T_{35}$. [2]

# SECTION B

Answer any THREE questions from section B.

**B2.** A parent sets up a fund for their son by making monthly deposits. If he deposits $E100$, $E105$, $E110$ at the end of the first, second, and third months, respectively, (the amounts increasing by $E5$ every month), find

(a) the instalment after 1 year. [8]

(b) the time when the instalment will reach $E500$. [6]

(c) the total deposited after 2 years. [6]

**B3.** Use Cramer's rule to solve the following linear system of equations.

$$\begin{aligned} x + 2y + z &= 3, \\ 2x + 4y + z &= 3, \\ 3x + 7y &= 2. \end{aligned}$$

[20]

**B4.** (a) Given the expression $(2x + y)^{18}$, Find

i. the first 4 terms. [8]

ii. the 14th term . [3]

iii. the middle term. [3]

(b) Solve $\log_3(x - 3) + \log_3(x + 3) = 3$. [6]

**B5.** (a) Find the equation of the line parallel to the line $6x - 2y = 1$ and passes through the point $(3, 4)$. [6]

(b) Find the amount at the end of 5 years on an original principal of $E5000$ at 6% if the interest is

i. simple interest. [4]

ii. compounded semiannually. [5]

iii. compounded continuously. [5]

**B6.** (a) A sum of $E16,000$ is invested for 5 years at 8% compound interest which is paid contin- uously. The total value is subsequently given by

$$V(t) = 16000e^{0.08t}$$

where $t$ is the number of years after the initial investment. Find

i. the total amount after 5 years. [2]

ii. how long it takes for the total amount to be double the initial investment? [6]

(b) Solve

i. $7^{x-3} = 1$. [4]

ii. $e^{1-x} = \frac{1}{2}$. [4]

iii. $\ln(4x + 17) = 0$. [4]

**END OF EXAMINATION**

# Formula Sheet

**Arithmetic Progressions:**

$$T_n = T_1 + (n-1)d, \quad S_n = \frac{n}{2}[T_1 + T_n], \quad S_n = \frac{n}{2}[2T_1 + (n-1)d].$$

**Geometric Progressions:**

$$T_n = T_1 r^{n-1}, \quad S_n = \frac{T_1(1-r^n)}{1-r}.$$

**Binomial Theorem:**

$$(a+b)^n = a^n + nC_1 a^{n-1}b + nC_2 a^{n-2}b^2 + nC_3 a^{n-3}b^3 + \cdots + b^n.$$
$$r\text{th term of } (a+b)^n = nC_{r-1} a^{n-r+1} b^{r-1}.$$

**Matrices:**

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

**logarithms and Exponential Functions:**

$$\log_b x = y \Leftrightarrow x = b^y.$$
$$\log_b(AB) = \log_b A + \log_b B.$$
$$\log_b\left(\frac{A}{B}\right) = \log_b A - \log_b B.$$
$$\log_b A^n = n \log_b A.$$

**The Quadratic Formula:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

**Simple Interest:**

$$A = P(1 + rn)$$

**Compound Interest:**

$$A = P(1+r)^n, \quad A = P(1 + r/s)^{ns}$$

**Continuous Compound Interest:**

$$A = Pe^{rn}$$