

UNIVERSITY OF SWAZILAND
FINAL EXAMINATION, 2017/2018
BASS I

Title of Paper : Elementary Quantitative Techniques II

Course Number : MAT102

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
 - a. SECTION A (40 MARKS)
 - Answer ALL questions in Section A.
 - b. SECTION B
 - There are FIVE (5) questions in Section B.
 - Each question in Section B is worth 20 Marks.
 - Answer ANY THREE (3) questions in Section B.
 - If you answer more than three (3) questions in Section B, only the first three questions answered in Section B will be marked.

2. Show all your working.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A

ANSWER ALL QUESTIONS

QUESTION A1

(a) Use the limit definition to find $f'(x)$

(i) $f(x) = x^2 + 1$ (5 marks)

(ii) $f(x) = \sqrt{2x + 1}$ (5 marks)

(b) Find the derivatives of the following:

(i) $y = x^2 \ln x$ (5 marks)

(ii) $y = \frac{\sin 2x}{1 + 4x^2}$ (5 marks)

QUESTION A2

Differentiate the following:

(i) $y = 3x^2 - 10x^{-\frac{1}{5}} + 4$ (5 marks)

(ii) $y = (7x^2 + 3x - 2)(x^4 + 1)$ (5 marks)

(iii) $y = 3 \sec(3x^2 + 5)$ (5 marks)

(iv) $y = x^{\sin x}$ (5 marks)

SECTION B

ANSWER ANY THREE QUESTIONS

QUESTION B3

- (a) A company manufactures and sells x radios per week. If the weekly cost and price-demand functions are given by

$$C(x) = 60000 + 72x, \quad p(x) = \frac{700}{3} - \frac{x}{36}$$

Find the following, for each week:

- (i) the cost of producing the 51st radio (2 marks)
 - (ii) the number of radios that maximise profit (4 marks)
 - (iii) the maximum profit (4 marks)
- (b) Sketch the graph of $y = 2x^3 - 21x + 60x + 10$, showing all relative extrema intervals where the curve is increasing/decreasing. (10 marks)

QUESTION B4

- (a) Find the first four (4) derivatives of the function $y = \ln \sqrt{2x + 1}$. (8 marks)
- (b) From a thin piece of cardboard that is 12cm by 12cm , square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? (12 marks)

QUESTION B5

Evaluate the following integrals:

- (a) $\int \left(2x - 3x^2 + \frac{4}{x^3} \right) dx$ (5 marks)
- (b) $\int x^2 e^x dx$ (5 marks)
- (c) $\int \left(\frac{5x - 7}{x^2 - 2x - 3} \right) dx$ (5 marks)

(d) $\int x^2(2x^3 + 1)^4 dx$ (5 marks)

QUESTION B6

(a) Find the area bounded by the curves $y = x^2 - 7x$ and $y = 9x - x^2$ (10 marks)

(b) Use integration by parts to evaluate

$$\int x^2 \cos 3x dx \quad (10 \text{ marks})$$

QUESTION B7

(a) A computer firm is marketing a view computer model. It determines that in order to sell x computers, the price per computer must be $p = 280 - 0.4x$. It also determines that the total cost of producing x computers is given by

$$C(x) = 5000 - 0.6x^2. \text{ Find the}$$

- (i) marginal average cost function
- (ii) marginal average profit function.

(b) Evaluate the following definite integrals:

(i) $\int_2^4 \frac{x}{2 + 5x^2} dx$

(ii) $\int_{1/2}^{e/2} \frac{\ln(2x)}{x} dx$

(10 marks)