# UNIVERSITY OF SWAZILAND

## SUPPLEMENTARY EXAMINATION, 2017/2018

## BASS I

Title of Paper : Elementary Quantitative Techniques II

Course Number : MAT102

Time Allowed : Three ( 3) Hours

Instructions

1. This paper consists of TWO ( 2) Sections:

   a. SECTION A ( 40 MARKS)
      - Answer ALL questions in Section A.

   b. SECTION B
      - There are FIVE ( 5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer ANY THREE ( 3) questions in Section B.
      - If you answer more than three ( 3) questions in Section B, only the first three questions answered in Section B will be marked.

2. Show all your working.

Special Requirements: NONE

**THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.**

## SECTION A

## ANSWER ALL QUESTIONS

### QUESTION A1

(a) Use the limit definition to find $f'(x)$

$$y = \frac{1}{x+1}$$ (4 marks)

(b) Find the equation for the tangent to the curve at the given point:

$$f(x) = x^3 - 3x \quad \text{at} \quad p(2,2)$$ (4 marks)

(c) Find the derivative of $y$ with respect to $x$ for $y = x^x$ (4 marks)

(d) Find the first and second derivative of the following:

$$f(n)\, x^2 \cos x$$ (8 marks)

### QUESTION A2

(a) Find $dy/dx$ for the following, in as simplified a form as possible

(i) $y = (x^2+1)^{3/2} + \sqrt{x^2+1}$ (5 marks)

(ii) $y^3 + y + x^2 + x = 0$ (5 marks)

(b) Evaluate the following integrals:

(i) $\int \frac{x}{(x-1)^2(X+1)}\, dx$ (5 marks)

(ii) $\int x(\ln x)^2\, dx$ (5 marks)

## SECTION B

## ANSWER ANY THREE QUESTIONS

### QUESTION B3

(a) Evaluate the following integral:

$$\int \frac{dx}{\left(4+x^2\right)^2}$$ (6 marks)

(b) Find the area bounded by the curve $y = xn - x^2$ and the straight line $y = 3x$. (7 marks)

(c) Use partial fractions to evaluate

$$\int \frac{x^2+x+1}{x^3-x^2-x+1}dx$$ (7 marks)

### QUESTION B4

(a) Evaluate

(i) $\int \sin^4 x \cos^3 x \, dx$ (5 marks)

(ii) $\int \frac{x^2}{\left(9-x^2\right)^{3/2}}dx$ (5 marks)

(b) Find $\frac{dy}{dx}$ for $y = x^2 \sin\left(\frac{1}{x}\right)$ (5 marks)

(c) Find $\frac{d^3y}{dx^3}$ for $y = x^5 \ln x$ (5 marks)

### QUESTION B5

(a) Consider the function

$$y = 4 + 27x - x^3$$

(i) Find all relative maxima and/or all relative minima (6 marks)

(ii) Find the $y-$ intercept (2 marks)

(iii) Make a sketch of the graph. (4 marks)

(b) The profit of a company is given by $p(x) = 75x - 0.015x^2 - 10000$ (in emalangeni) where $x$ is the number of units sold per month. Find the

(i) profit if the number of units sold is 1500. (3 marks)

(ii) maximum monthly profit. (5 marks)

**QUESTION B6**

(a) Find the area of the region bounded by the parabola $y = -x^2 - 6x$ and the line $y = 0$ (8 marks)

(b) Find the equation of the curve that passes through (2,5) if its slope is given by $\frac{dy}{dx} = 2x$ at any point $x$. (6 marks)

(c) If the marginal cost of producing $x$ units is given by

$$C'(x) = 0.3x^2 + 2x$$

and the fixed cost is E2000, find the cost function $C(x)$ (6 marks)

**QUESTION B7**

Given the demand function $D(x) = 20 - 0.05x$ and the supply function $S(x) = 2 + 0.0002x^2$, find

(a) the equilibrium price (6 marks)

(b) the consumer's surplus (7 marks)

(c) the producer's surplus (7 marks)