University of Swaziland

Main Examination, 2017/2018

BASS I , BCom I, BEd I, BCom I(IDE), BEd I(IDE)

Title of Paper : Calculus for Business Studies

Course Number : MAT108/MS102

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. Indicate your program next to your student ID and indicate whether you are **full time student** or **part time student**.

Special Requirements: NONE

This examination paper should not be opened until permission has been given by the invigilator.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

i) Evaluate the following limits

(a) $\lim_{x \to 1} \frac{x^2 - x}{x - 1}$. [3]

(b) $\lim_{x \to \infty} \frac{2x^2 - x + 1}{1 - 3x^2}$. [3]

(c) $\lim_{x \to 0} \frac{x^3 - 2x^2}{x^4 + 3x^2}$. [3]

ii) Consider the function $f(x) = \frac{x^2 - 1}{x + 1}$. Find the point of discontinuity, hence classify the type of discontinuity. [3]

iii) Consider the function $y = 2x^3 + 3x + 1$.

(a) Find the first derivative $y'(x)$. [2]

(b) Hence find the equation of the tangent line to the curve at $x = 1$. [3]

iv) Find the derivatives of the following functions

(a) $y = (x^2 + 1)e^x$ [4]

(b) $y = \frac{\ln(x)}{x^2}$ [4]

(c) $y = (4x^3 - 7)^9$ [4]

v) Integrate the following functions

(a) $\int \left(\frac{7}{x} + 3x + 2e^x + 4 \right) dx$ [3]

(b) $\int 2x(x^2 - 2)^8 dx$ [3]

(c) $\int_0^{90^\circ} x \cos(3x) dx$ [5]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

a) Use the limit definition of the derivative to find the derivative, $f'(x)$ of the function [10]

$$f(x) = 4x^2 - 8x + 12.$$

b) Consider the function defined by $f(x) = \begin{cases} 2x, & \text{if } x \leq 5 \\ 3x + 5, & \text{if } x > 5 \end{cases}$.

i) Evaluate $f(5)$. [1]

ii) Find $\lim_{x \to 5^-} f(x)$ and $\lim_{x \to 5^+} f(x)$. [4]

iii) Find $\lim_{x \to 5} f(x)$ if it exists. [2]

iv) Is the function continuous at $x = 5$? Explain. [3]

QUESTION B3 [*20 Marks*]

a) Use logarithmic differentiation to find the derivative, y' given [5]

$$y = (x^2 + 1)^{\ln x}.$$

b) Find y''' given $y = e^{x^2}$. [5]

c) The total profit (in Emalangeni) from the sale of x watches is $P(x) = 20x - 0.02x^2 - 320$.

i) Find the average profit per watch if 40 watches are produced. [4]

ii) Find the marginal average profit at a production level of 40 watches and interpret. [6]

QUESTION B4 [*20 Marks*]

Consider the function $y = x^3 - 12x + 12$.

a) Find all critical values. [3]

b) Find intervals of increase and decrease. [6]

c) Find all possible inflection points. [2]

d) Find the intervals where the curve is concave up and concave down. [4]

e) Sketch the curve showing clearly, all points of inflection, relative maximum or minimum, $y-$ intercepts and $x-$ intercepts where applicable. [5]

QUESTION B5 [*20 Marks*]

a) A computer firm is marketing a new computer model. It determines that in order to sell x computers, the price per computer must be $p = 280 - 0.4x$. It also determines that the total cost of producing x computers is given by $C(x) = 5000 + 0.6x^2$.

i) Determine the revenue function $R(x)$. [2]

ii) Determine the profit function $P(x)$. [2]

iii) What price per computer must be charged in order to make maximum profit? [6]

b) The rate of growth of the population $N(t)$ of a new city t years after its incorporation is estimated to be [10]

$$\frac{dN}{dt} = 400 + 600\sqrt{t}.$$

If the population was 5000 at the time of incorporation, find the population 9 years later.

QUESTION B6 [*20 Marks*]

a) Integrate the following functions;

i) $\int xe^x dx.$ [3]

ii) $\int \frac{x-1}{x(4x+1)} dx.$ [5]

b) Consider the demand function $D(x) = 300 - 6x - x^2$ and the supply function $S(x) = x^2 + 4x$

i) Determine the equilibrium price and equilibrium quantity. [2]

ii) Find the consumer's surplus at the equilibrium price level. [5]

iii) Find the producer's surplus at the equilibrium price level. [5]

END OF EXAMINATION PAPER