
University of Swaziland



Final Examination – December 2017

BSc I, BEng I, BEd I, BASS I

Title of Paper : Algebra, Trigonometry and Analytic Geometry
Course Number : MAT111
Time Allowed : Three (3) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 3 (out of 5) questions in Section B.
4. Show all your working.
5. Begin each question on a new page.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 a. Solve for x

i. $\ln(2x - 19) = 0$ [2 marks]

ii. $7^{x+2} = 700$ (correct 2 d.p.) [2 marks]

b. Evaluate

$$\frac{5}{1+2i} + \frac{10}{2-i}$$

and leave your answer in the form $a + ib$. [5 marks]

c. Given that $\tan A = \sqrt{3}$ and $\cos A < 0$, find the *exact* value of

$$\sin A \tan A - \cos A. \quad [5 \text{ marks}]$$

d. Find the value of

$$\sum_{n=-10}^{100} (3n - 25). \quad [4 \text{ marks}]$$

e. Consider the equation of a circle

$$x^2 + y^2 = 8x.$$

i. Find the coordinates of the centre and the length of the radius [3 marks]

ii. Make a sketch of the circle. [2 marks]

f. Given the matrices

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -3 & 0 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 3 & -2 \\ 2 & -1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} -1 & 2 \\ 1 & -3 \end{pmatrix},$$

find

$$AB^T + 2C. \quad [5 \text{ marks}]$$

g. Use *synthetic division* to find the quotient and remainder of

$$\frac{x^4 - 6x^2 + 13}{x + 1}. \quad [5 \text{ marks}]$$

h. In the binomial expansion of

$$\left(x^2 - \frac{y^3}{x}\right)^{19},$$

find

i. the first 3 terms [5 marks]

ii. the 13th term [2 marks]

Section B

Answer ANY 3 Questions in this section

B.1 a. For the binomial expansion of

$$\left(\frac{1}{x} - 2x^2\right)^{-2}$$

in ascending powers of x , find

- i. the first 3 terms [3 marks]
- ii. the term involving x^{23} . [5 marks]

b. For the hyperbola defined by the equation

$$x^2 - 4y^2 + 6x + 16y = 11,$$

- i. transform the equation into *standard form* [3 marks]
 - ii. find the coordinates of the centre, vertices and foci [3 marks]
 - iii. find the equations of the asymptotes [2 marks]
 - iv. find the eccentricity [1 mark]
 - v. make a sketch of the hyperbola [3 marks]
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B.2 a. *Without using a calculator*, find the *exact* value of

- i. $\sin\left(\frac{29}{12}\pi\right)$ [3 marks]
- ii. $\left(\cos 525^\circ - \sin 525^\circ\right)^2$ [3 marks]

showing all your steps.

b. Given that $\sin \theta = \lambda > 0$ while $\tan \theta < 0$, find

- i. $\tan^2 \theta + \sec^2 \theta$ [4 marks]
- ii. $\cos 2\theta$ [3 marks]

in terms of λ .

c. Prove each of the following trigonometric identities:

- i. $\sec^4 \theta - 2 \tan^2 \theta - \tan^4 \theta = 1$ [3 marks]
 - ii. $\frac{\sin 3\theta - \sin \theta}{\cos^2 \theta - \sin^2 \theta} = 2 \sin \theta$ [4 marks]
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B.5 a. The vertices of a triangle in 3-D space are $A(2, 0, -5)$, $B(0, 4, 2)$ and $C(0, 0, 8)$.

Find the

- i. length of the side AB [2 marks]
- ii. the interior angle \hat{ACB} [6 marks]
- iii. the area of the triangle [3 marks]

b. Consider the complex number

$$\Omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}.$$

- i. Express Ω in polar form [3 marks]
- ii. Use de Moivre's theorem to find Ω^{20} , leaving your answer in the form $a + ib$. [3 marks]
- iii. Find the square roots of Ω , leaving your answer in the form $a + ib$. [4 marks]

END OF EXAMINATION
