University of Swaziland

Re-sit Examination – July 2018

BSc I, BEng I, BEd I, BASS I

Title of Paper : Algebra, Trigonometry & Analytic Geometry
Course Number : MAT111
Time Allowed : Three (3) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 3 (out of 5) questions in Section B.
4. Show all your working.
5. Begin each question on a new page.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN
BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 a. Find the value of

  i. $\log_{\frac{1}{2}} 425$ (correct to 2 d.p.) [2 marks]

  ii. $\log_b b^4 + \ln e^{4m-3}$ [4 marks]

b. Evaluate
$$(2 + 3i)^2 - (2 - 3i)^2.$$

and leave your answer in the form $a + ib$. [5 marks]

c. Expand and simplify

$$\left(\cos A + \sin A \right)^2 + \left(\cos A - \sin A \right)^2.$$ [5 marks]

d. Find the value of

$$\sum_{n=0}^{\infty} 32 \left(\tfrac{3}{7} \right)^n.$$ [4 marks]

e. Consider the equation of a parabola

$$y^2 + 12x = 0.$$

  i. Find the coordinates of the vertex and focus [3 marks]

  ii. Make a sketch of the parabola. [2 marks]

f. Evaluate the determinant
$$\begin{vmatrix} 2 & -1 & 1 \\ -3 & 0 & -2 \\ 4 & 2 & 0 \end{vmatrix}.$$ [5 marks]

g. Find the quotient and remainder of

$$\frac{x^4 - 6x^3 + 8x + 13}{x^2 + 1}.$$ [5 marks]

h. In the binomial expansion of

$$\left(x + \frac{2}{x} \right)^{24},$$

find the first 3 terms. [5 marks]

Section B

Answer ANY 3 Questions in this section

B.1 a. If the first 3 terms in the binomial expansion of $(1 + kx)^n$ are $1 - 6x + \frac{33}{2}x^2$, find the values of k and n. [4 marks]

b. Consider the triangle with vertices $A(4, 9)$, $B(7, -6)$ and $C(-4, 1)$. Find

    i. the equation of side AB, expressing it in *general form* [4 marks]

    ii. the *perpendicular* distance from vertec C to side AB [3 marks]

    iii. the interior angle $\hat{A}$ [3 marks]

    iv. the perimeter of the triangle (correct to 2 d.p.) [3 marks]

    v. the area of the triangle (correct to 2 d.p.) [3 marks]

B.2 a. Simplify
$$\sin\left(\tfrac{1}{6}\pi + A\right) + \cos\left(\tfrac{1}{3}\pi + A\right).$$ [4 marks]

b. Find the *general solution* of the equation
$$2\sin^2\theta = 1 + \cos\theta,$$
expressing your answer in radians. [7 marks]

c. Prove each of the following trigonometric identities:

    i. $\cos^4\theta + \sin^2\theta\cos^2\theta - \cos^2\theta = 0$ [3 marks]

    ii. $\dfrac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \tan\theta$ [6 marks]

B.3 a. Solve for x given

   i. $\ln(90x + 5) - \ln x = \ln(10x) + \ln\left(\dfrac{10}{x}\right)$ [4 marks]

   ii. $\log_2 x + \log_2(x - 2) = \log_b b^3$.   [4 marks]

b. Express as a single logarithm with unit coefficient, and simlify

$$3 + 2\log_2\left(2a^2\right) - 4\log_2\left(2a\right).$$ [4 marks]

c. On 01 January 2017, a sum of E10,000 is invested in an account that pays 6.7% p.a. compouded daily. Find

   i. the total accrued on 30 June 2022   [3 marks]

   ii. the *date* on which the total with be triple the initial sum.   [5 marks]

B.4 a. Find the quotient and remainder of

$$\frac{y^4 + x^4}{y + x}.$$ [5 marks]

b. Factorise the polynomial

$$P(x) = 6x^3 + 19x^2 + 2x - 3.$$ [5 marks]

b. Use mathematical induction to prove the formula

$$\sum_{i=0}^{n} \rho^i = \frac{1 - \rho^{n+1}}{1 - \rho}. \quad \rho \neq 1, \; n \in \mathbb{Z}^+.$$ [10 marks]

B.5 a. Given the vectors $A = 2\hat{\imath} - 5\hat{k}$ and $B = -8\hat{\jmath} + 2\hat{k}$, find

   i. the angle made by the vectors A and B [5 marks]

   ii. the cross product

$$A \times B.$$ [5 marks]

b. Find the *exact* value of

$$\left(-1 + 2i\right)^{14}.$$ [4 marks]

c. Given that $2 - i$ is a root of the polynomial

$$P(z) = 3z^3 - 10z^2 + 7z + 10,$$

find the other roots.   [6 marks]

END OF EXAMINATION