University of Swaziland

Supplementary Examination, July 2018

B.Sc II, B.A.S.S II, B.Ed II, B.Eng II

Title of Paper : Calculus I

Course Code : MAT211/M211

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.

   a. **SECTION A(COMPULSORY): 40 MARKS**
   Answer ALL QUESTIONS.

   b. **SECTION B: 60 MARKS**
   Answer ANY THREE questions.
   Submit solutions to ONLY THREE questions in Section B.

2. Each question in Section B is worth 20%.

3. Show all your working.

4. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) (i) State the Mean Value Theorem for derivatives. [2]

(ii) Determine the absolute extrema for the function, $f(x) = 2x^3 + 3x^2 - 12x + 4$ on [-4, 2]. [4]

(iii) Find the intervals where the function $f(x) = x + \frac{1}{x}$ is increasing and where it is decreasing. [4]

(b) (i) State the Mean Value Theorem for Integrals. [3]

(ii) Find the area of the region bounded above by the curves $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \frac{\pi}{2}$. [4]

(c) (i) State the Ratio Test for series convergence. [3]

(ii) Test the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ for Convergence or Divergence. [5]

(iii) When do we say that a given series is conditionally convergent. Explain your answer. [3]

(iv) Show that the sequence $< \frac{3n+7}{4n+8} >$ is monotonically decreasing. [4]

(v) Evaluate $\lim_{x \to 0} \frac{1 - \cos x}{x \ln(1+x)}$. [5]

(vi) Discuss the boundedness of the sequence $< (-1) \cdot 5 >$. [3]

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

(a) Verify Rolle's theorem for the function $f(x) = 2 + (x-1)^{2/3}$ in the interval $[0, 2]$. [6]

(b) Verify that the function $f(x) = x^3 - 6x^2 + 11x - 5$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 4]$. [6]

(c) An open cylindrical can of given capacity is to be made from a metal sheet of uniform thickness. If no allowance is to be made for waste of material, what will be the most economical ratio of the radius to the height of the can? [8]

Question 3

(b) Evaluate the following limits.

(i) $\lim_{x \to 0} \dfrac{1 - \sqrt{1 - x^2}}{x^2}$ [4]

(ii) $\lim_{x \to 0} \dfrac{\ln x}{\cot x}$ [4]

(iii) $\lim_{x \to 0^+} x^x$ [4]

(c) Sketch the graph of the function $y = x^4 - 2x^2 + 7$. (Show the necessary details) [8]

Question 4

(a) Determine the number c that satisfies the Mean Value Theorem for Integrals for the function $f(x) = x^2 + 3x + 2$ on the interval $[1, 4]$. [8]

(b) Let B be a number > 1. What is the volume of solid generated by the area under the curve $y = e^{-x}$ between 1 and B (the axis of revolution being the $x-$axis)? Does the volume approach a limit as B become large? If so, to what limit? [12]

Question 5

(a) Prove that the sequences, $< a_n >$ where: $a_n = \dfrac{n+1}{n}$ is monotonic. Find out whether it is increasing or decreasing. [6]

(b) (i) True or false: Every bounded sequence is convergent. Explain your answer. [3]

(ii) Prove that the sequence $a_n = \dfrac{2n+3}{3n+4}$ is bounded. [5]

(c) Examine the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n}{n+1}.$$

[6]

Question 6

(a) Using Integral Test determine the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$. [10]

(b) Expand $\ln \sin x$ in powers of $(x-3)$ up to the first four terms. [10]

End of Examination Paper