University of Swaziland

Final Examination, 2017/2018

B.Sc. II, B.Ed II

Title of Paper : Mathematics for Scientists

Course Number : M215/MAT215

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.
   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked**.
2. Show all your working.

Special Requirements: None

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: Answer ALL Questions

A1. Do the points $(-1.1), (5, -3)$. and $(2, -1)$ lie on one line? (4)

A2. Find all numbers m such that the vectors $\overline{a} = (1, 5m, 6m)$ and $\overline{b} = (-1, -1, m)$ are orthogonal. (4)

A3. For the given function $f(x)$ find all numbers x_i in the open interval (a, b) for which mean value theorem is satisfied if

(a) $f(x) = x^2 + 1; (a, b) = (1, 2)$.

(b) $f(x) = \frac{x-1}{x}; (a, b) = (1, 3)$. (4,4)

A4. Find the third Taylor polynomial at $x_0 = 0$, for $f(x) = \frac{1}{x+1}$. (4)

A5. Verify equality of mixed derivatives theorem for $f(x, y) = \sin(2x + 3y)$. (4)

A6. Minimize $f(x, y) = 2x^2 + y^2$, subject to constraints $x + 2y = 3$. (5)

A7. Compute the volume under the graph of $z = 3x^2y + 1$ over the region $1 < x < 2, 1 < y < 3$. (5)

A8. Solve the following initial value problem

$$\frac{dy}{dx} = 5y, \quad y(0) = y_0$$

(6)

SECTION B: Answer Any *THREE* Questions

QUESTION B1 [*20 Marks*]

B1. (a) Apply vector product to find

(i) a unit vector perpendicular to both

$\overline{a} = (2, -6, -3)$ and $\overline{b} = (4, 3, -1)$.

(ii) Area of the parallelogram spanned by the vectors $\overline{a} = (2, 3, -1)$ and $\overline{b} = (1, 2, -4)$.

(5,5)

(b) Find the volume of parallelepiped spanned by the directed segments $\overline{OA}, \overline{OB}$, and $\overline{OC}$, if the coordinates of A, B, and C are $(1, 1, 1), (0, 1, 1)$ and $(1, 0, 4)$, respectively. (5)

(c) Given Rolle's theorem, state and prove mean value theorem. (5)

QUESTION B2 [*20 Marks*]

B2. (a) Evaluate

(i) $\lim_{x\to 0} \frac{\sin x - e^x + 1}{x^2}$,

(ii) $\lim_{x\to\infty} \frac{e^x + lnx}{e^x + x}$, (4,4)

(b) (i) State Taylor's theorem.

(ii) Expand $\cos x$ in Taylor series for small $|x|$.

(iii) Use quadratic approximation to compute $\cos x$ and estimate the error.

(iv) In particular compute $\cos 0.2$ and estimate the error. (3,3,3,3)

QUESTION B3 [*20 Marks*]

B3. (a) Apply the chain rule to evaluate f'_u and f'_v if $f(x,y) = x^2 + y^2, x = u\cos v, y = u\sin v$. (6)

(b) Find and classify all stationary points of $f(x,y) = 2x^2 + y^2 - 2x + y$. (4)

(c) A container manufacturer is designing a closed rectangular box, having square base and a volume $1024m^3$. The tops and bottoms must be stronger (to put on top of each other). Material for the top and bottom costs $4\frac{E}{m^2}$, for sides $2\frac{E}{m^2}$. Find dimensions which minimize total cost. (10)

QUESTION B4 [*20 Marks*]

B4. (a) Compute the volume of solid under surface $z = \sqrt{x}y$ and over the set of points (x,y) with $x > 0, y > x^2, y < 2 - x^2$. (7)

(b) Pass to polar coordinates to evaluate a double integral of $f(x,y) = \frac{y}{\sqrt{x^2+y^2}}$ over the region R, which is in the upper half-plane bounded by the circle $x^2 + y^2 = 16$ and the x-axis. (6)

(c) Compute a triple integral of $x^2 + y^2 + z^2$ over a region D, where D is a cube $0 < x < 1, 0 < y < 1, 0 < z < 1$. (7)

QUESTION B5 [*20 Marks*]

B5. (a) Solve the ODE with homogeneous coefficients

$$(x^2 - xy + y^2)dx - xydy = 0$$

(6)

(b) Test the ODE for exactness and solve it

$$(2t^3x - 5x^4)\frac{dx}{dt} = -3t^2x^2 - 1.$$

(7)

(c) Solve the initial value problem

$$2y'' + 5y' - 3y = 0, y(0) = 7, y'(0) = 0.$$

(7)

END OF EXAMINATION PAPER