University of Swaziland

Supplementary Examination, 2017/2018

B.Sc. II, BEd. II

Title of Paper : Mathematics for Scientists

Course Number : M215/MAT 215

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.
   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked**.
2. Show all your working.

Special Requirements: None

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: Answer ALL Questions

A1. Do the points $\left(\frac{1}{2}, \frac{1}{2}\right)$, $\left(1, \frac{3}{2}\right)$ and $\left(\frac{5}{2}, 4\right)$ lie on one line? (4)

A2. Find the angle between $\bar{a} = (1, -1, 0)$ and $\bar{b} = (0, 1, -1)$. (4)

A3. For the given function $f(x)$ find all numbers x_i in the open interval (a, b) for which mean value theorem is satisfied, if

(a) $f(x) = x^3 - x, (a, b) = (0, 1)$.

(b) $f(x) = 1 - x^{\frac{2}{3}}, (a, b) = (-1, 1)$. (4,4)

A4. Express the polynomial $p(x) = 7 - 8x + 4x^2 + 5x^3 - 3x^4$ in Taylor form about $x_0 = 1$. (4)

A5. Verify equality of mixed derivatives theorem for $f(x, y) = x^2y + xy^3$. (4)

A6. Minimize $f(x, y) = x^2 + y^2$, subject to constraint $x + 2y = 3$. (5)

A7. Compute the volume under the graph of $z = 6x^2y + 8xy^3$ over the region $1 < x < 2, 3 < y < 4$. (5)

A8. Solve the following initial value problem

$$\frac{dy}{dx} = -4yx, \quad y(0) = y_0$$

(6)

SECTION B: Answer Any *THREE* Questions

QUESTION B1 [*20 Marks*]

B1. (a) Use cross product to find

(i) a unit vector perpendicular to both

$$\bar{a} = i + 2k, \bar{b} = i + j - k.$$

(ii) The area of the triangle spanned by the vectors $\bar{a} = (0, 2, 4), \bar{b} = (2, 4, 2)$. (5,5)

(b) Find the volume of parallelepiped spanned by the directed segments $\overline{OA}, \overline{OB}$, and $\overline{OC}$ if the coordinates of A, B, and C are $(1, 0, 0), (1, 1, 0)$ and $(0, 0, 4)$, respectively. (5)

(c) (i) State and prove the Rolle's theorem, and thus

(ii) Apply it for $f(x) = |x| - 1, -1 < x < 1$. (5)

QUESTION B2 [*20 Marks*]

B2. (a) Evaluate

(i) $\lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{\cos x}$;

(ii) $\lim_{x\to 0} \frac{\tan x - x}{x - \sin x}$. (4,4)

(b) (i) Write down Maclaurin's formula with its reminder.

(ii) Expand $\sin x$ in Maclaurin's series.

(iii) Use quadratic approximation to compute $\sin x$, and estimate the error for small $|x|$.

(iv) In particular, compute $\sin 0.06$ and estimate the error. (3,3,3,3)

QUESTION B3 [*20 Marks*]

B3. (a) Apply the chain rule to evaluate f'_u and f'_v if $f(x,y) = x^2 - y^2, x = u^2 + v^2, y = 2uv$. (6)

(b) Find and classify all stationary points of $f(x,y) = x^2 - y^2 + 4x + 2y$. (4)

(c) An experimental farm has 3600m of fencing to create a rectangular area that will be subdivided into four equal subregions. What is the largest total area that can be enclosed? (10)

QUESTION B4 [*20 Marks*]

B4. (a) Compute the volume of solid under surface $z = x^2y$ and over the interior of the triangle with vertices $(0,0), (0,1), (1,0)$. (6)

(b) Pass to polar coordinates to evaluate a double integral of $\exp(x^2+y^2)$ over the region D, where

$$D = \{(x,y) : x > 0, y > 0, 1 < x^2 + y^2 < 9\}.$$

(6)

(c) Compute a triple integral of $f(x,y,z) = yz^2\exp(xyz)$ over a region D, where D is a cube $0 < x < 1, \quad 0 < y < 1, \quad 0 < z < 1$. (8)

QUESTION B5 [*20 Marks*]

B5. (a) Solve ODE with homogeneous coefficients

$$2(2x^2 + y^2)dx - xydy = 0.$$

(6)

(b) Test the ODE for exactness and solve it $3x(xy-2)dx + (x^3 + 2y)dy = 0$. (7)

(c) Solve the initial value problem

$$y'' + 4y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = -1.$$

(7)

END OF EXAMINATION PAPER