University of Swaziland

Main Examination, 2017/2018

BASS II, B.Ed (Sec.) II, B.Sc. II, B.Eng. II

Title of Paper : Ordinary Differential Equations

Course Number : MAT216/M213

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

a) Solve the following differential equation [6]

$$2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0.$$

b) Solve the following differential equation [7]

$$y'' - 8y' + 17y = 0.$$

c) By eliminating the constants, determine the ODE satisfied by the function [5]

$$y = (A + xB)e^{-2x}$$

d) Find the particular solution of the ordinary differential equation [5]

$$y'' + 3y' - 28y = e^{-7t}$$

e) Reduce the ordinary differential equation into a system of first order ordinary differential equations, leaving your answer in matrix form. [5]

$$y''' + 8y'' + 16y = 0$$

f) Find the inverse Laplace transform of [7]

$$H(s) = \frac{s+7}{s^2 - 3s - 10}$$

g) Find the general solution of [5]

$$x^2y'' - 7xy' + 16y = 0.$$

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

a) Solve the initial value problem [6]

$$6y' - 2y = xy^4, \qquad y(0) = -2.$$

b) Consider the ordinary differential equation

$$(x\sin(y) + \cos(y))dy + (x+y)\sin(y)dx = 0$$

i) Show that the ordinary differential equation is not exact. [2]

ii) Find the solution of the ordinary differential equation. [6]

c) Consider the ordinary differential equation

$$(x^2 - y^2)dx + xydy = 0$$

i) Show that the ordinary differential equation is homogeneous. [2]

ii) Hence find the solution of the ordinary differential equation. [4]

QUESTION B3 [*20 Marks*]

a) Using the method of variation of parameters, find a general solution of the differential equation [12]

$$y'' - 2y' + y = e^x \ln(x).$$

b) Given that $y_1(x) = x$ is a solution of $x^2y'' - xy' + y = 0$,

i) Find a second linearly independent solution $y_2(x)$. [5]

ii) Show that $y_1(x)$ and $y_2(x)$ are linearly independent. [3]

QUESTION B4 [*20 Marks*]

a) Using Laplace Transforms, find the solution of the IVP [10]

$$2y'' + 3y' - 2y = te^{-2t}, \quad y(0) = 0, \quad y'(0) = -2.$$

b) Show that [10]

$$\mathcal{L}\{u''\} = s^2U(s) - su(0) - u'(0),$$

where $\mathcal{L}$ is the Laplace transform.

QUESTION B5 [*20 Marks*]

Solve the following ODE using the series solution method around $x_0 = 0$. Find the first four terms in each portion of the series solution.

$$(x^2 + 1)y'' - 4xy' + 6y = 0.$$

[20]

QUESTION B6 [*20 Marks*]

a) Find the general solution of the ordinary differential equation [6]

$$x^2y'' - 3xy' + 4y = 0.$$

b) Using the method of undetermined coefficients, find the general solution of the ordinary differential equation [6]

$$y'' + y = \cos(t).$$

c) Find the general solution of [8]

$$\frac{d\mathbf{X}}{dt} = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix} \mathbf{X},$$

where $\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}$. Note that x and y are functions of t.

END OF EXAMINATION PAPER

$f(t)$	$f(t) = F(s)$	$f(t)$	$f(t) = F(s)$
1	$\frac{1}{s}$	$\frac{ae^{at} - be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}$
$e^{at}f(t)$	$F(s-a)$	te^{at}	$\frac{1}{(s-a)^2}$
$\mathcal{U}(t-a)$	$\frac{e^{-as}}{s}$	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$f(t-a)\mathcal{U}(t-a)$	$e^{-as}F(s)$		
$\delta(t)$	1	$e^{at}\sin kt$	$\frac{k}{(s-a)^2+k^2}$
$\delta(t-t_0)$	e^{-st_0}	$e^{at}\cos kt$	$\frac{s-a}{(s-a)^2+k^2}$
$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{ds^n}$	$e^{at}\sinh kt$	$\frac{k}{(s-a)^2-k^2}$
$f'(t)$	$sF(s) - f(0)$		
$f^n(t)$	$s^n F(s) - s^{(n-1)}f(0) - \cdots - f^{(n-1)}(0)$	$e^{at}\cosh kt$	$\frac{s-a}{(s-a)^2-k^2}$
		$t\sin kt$	$\frac{2ks}{(s^2+k^2)^2}$
$\int_0^t f(x)g(t-x)dx$	$F(s)G(s)$	$t\cos kt$	$\frac{s^2-k^2}{(s^2+k^2)^2}$
$t^n \ (n = 0, 1, 2, \dots)$	$\frac{n!}{s^{n+1}}$	$t\sinh kt$	$\frac{2ks}{(s^2-k^2)^2}$
$t^x \ (x \geq -1 \in \mathbb{R})$	$\frac{\Gamma(x+1)}{s^{x+1}}$	$t\cosh kt$	$\frac{s^2+k^2}{(s^2-k^2)^2}$
$\sin kt$	$\frac{k}{s^2+k^2}$	$\frac{\sin at}{t}$	$\arctan \frac{a}{s}$
$\cos kt$	$\frac{s}{s^2+k^2}$	$\frac{1}{\sqrt{\pi t}} e^{-a^2/4t}$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$
e^{at}	$\frac{1}{s-a}$	$\frac{a}{2\sqrt{\pi t^3}} e^{-a^2/4t}$	$e^{-a\sqrt{s}}$
$\sinh kt$	$\frac{k}{s^2-k^2}$	$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{s}$
$\cosh kt$	$\frac{s}{s^2-k^2}$		
$\frac{e^{at} - e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}$		