UNIVERSITY OF SWAZILAND

RE-SIT/SUPPLEMENTARY EXAMINATION, 2017/2018

BASS II, B.Ed (Sec.) II, B.Sc. II

Title of Paper : Foundations of Mathematics

Course Number : MAT231/M231

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SEVEN (7) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, A2, B2, ..., B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*20 Marks*]

(a) Define each of the following.

i. A *proposition*. (2)

ii. A *contradiction*. (2)

iii. An *equivalence relation* on a set A. (3)

iv. A *partial order* on a set A. (3)

(b) Write down (i.) the inverse, (ii.) the converse, and (iii.) the contrapositive of the statement, "If I go to work, then I will earn money." (6)

(c) Let p be "It is cold" and let q be "It is raining". Use logical connectives to write the following statements symbolically. (4)

i. "It is not raining and it is cold."

ii. "It is raining if and only if it is cold."

iii. "Either it is raining or it is cold."

iv. "If it is raining, then it is cold."

QUESTION A2 [*20 Marks*]

(a) Let $A = \{1,2,3\}$ and $B = \{3,4\}$. Find

i. $B \times A$.

ii. $\mathscr{P}(A)$, the power set of A.

(3,3

(b) Let $D = \{1,2,3\}$ be the domain of discourse. Determine the truth value of each of the following propositions.

i. $(\exists x)(\forall y)(x^2 < y+1)$

ii. $(\forall x)(\exists y)(x^2 + y^2 < 12)$.

(3,3

(c) Negate the following propositions. (4,4

i. $(\forall x)(\neg p(x) \vee q(x))$

ii. $(\exists x)(\forall y)\left[p(x,y) \vee (\exists y)\neg q(y)\right]$.

END OF SECTION A – TURN OVER

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B3 [*20 Marks*]

(a) Construct a truth table for the proposition

$$(p \vee (p \to q)) \wedge \neg(q \wedge \neg r).$$

(4)

(b) Use a truth table to prove that the following argument is valid. (4)

$$\begin{aligned} &p \to q \\ &q \to r \\ \therefore\ &p \to r \end{aligned}$$

(c) Use fundamental logical equivalences (not truth tables) to prove the following logical equivalence. (4)

$$\neg p \vee (p \wedge q) \equiv p \to q.$$

(d) Consider the proposition

$$(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x < y).$$

i. Determine the truth value of the proposition. (2)

ii. Write down the negation of the proposition and determine its truth value. (3)

iii. Write down the proposition resulting from interchanging the symbols $\forall$ and $\exists$ and determine its truth value. (3)

QUESTION B4 [20 *Marks*]

(a) Prove: *If x^2 is even, then x is even.* (5)

(b) Prove: *For any integer n, the number $(n^3 - n)$ is even.* (5)

(c) Prove: *There is no rational number x such that $x^2 = 2$.* (5)

(d) Let $a, b, c \in \mathbb{Z}$ with $c \neq 0$ Prove: *If $c \mid a$ and $c \mid b$, then $c \mid (a + b)$.* (5)

QUESTION B5 [20 *Marks*]

(a) Let A and B be sets in a universal set U. Prove each of the following.

i. $\varnothing \subseteq A$ (3)

ii. $(A \setminus B) \cap B = \varnothing$. (4)

iii. $(A \cap B)^c = A^c \cup B^c$. (6)

(b) Let $S = \{1, 2, \ldots, 9\}$. Determine which of the following collections of subsets of S form a partition of S. (Give reasons for your answers.) (7)

i. $\{\{1,3,5\}, \{2,6\}, \{4,8,9\}\}$

ii. $\{\{1,3,5\}, \{2,4,6,8\}, \{5,7,9\}\}$

iii. $\{\{1,3,5\}, \{2,4,6,8\}, \{7,9\}\}$

QUESTION B6 [20 *Marks*]

(a) Let $A = \{2,3,4\}$ and $B = \{3,4,5,6,7\}$. Define a relation R from A to B by aRb if $a \mid b$.

i. List the elements of R. (3)

ii. Find the domain of R. (1)

iii. Find the range of R. (2)

(b) Define a relation $\sim$ on $\mathbb{Z}$ by $a \sim b$ if $a \equiv b \pmod 5$.

i. Show that $\sim$ is an equivalence relation on $\mathbb{Z}$. (6)

ii. List the equivalence classes of $\mathbb{Z}$ given by $\sim$. (2)

(c) Show that the relation $\leq$ is a partial order relation on $\mathbb{R}$. (6)

QUESTION B7 [20 *Marks*]

(a) Use mathematical induction to prove that $3^{2n} + 7$ is divisible by 8 for all integers $n \geq 0$. (6)

(b) Use strong induction to prove: *Any integer $n > 1$ can be written as a product of prime numbers.* (7)

(c) Find an explicit formula for the Fibonacci sequence

$$a_1 = a_2 = 1, \quad a_n = a_{n-1} + a_{n-2}.$$

(7)

END OF EXAMINATION PAPER