University of Swaziland

Examination, 2017/2018

BASS, B.Ed (Sec.), B.Sc.

Title of Paper : Dynamics I

Course Number : MAT256/M255

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth a total of 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2, B3 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

(a) Let $\mathbf{a} = -\hat{\imath} + 2\hat{\jmath} + \hat{\mathbf{k}}$ and $\mathbf{b} = 2\hat{\imath} - 3\hat{\mathbf{k}}$. Find a unit vector perpendicular to both $\mathbf{c} = \mathbf{b} - \mathbf{a}$ and $\mathbf{d} = 2\mathbf{a} + \mathbf{b}$. (5)

(b) Let $\mathbf{F}(x, y, z) = 2xy^2\hat{\imath} + (2x^2y + 2yz^2)\hat{\jmath} + 2y^2z\hat{\mathbf{k}}$. Show that $\mathbf{F}$ is a conservative vector field and find a scalar potential for $\mathbf{F}$. (10)

(c) A particle moves along the space curve defined by

$$\mathbf{r}(t) = A\cos\omega t\hat{\imath} + A\sin\omega t\hat{\jmath},$$

where A and ω are positive constants. Find

i. the velocity of the particle, (2)
ii. the speed of the particle, (2)
iii. the acceleration of the particle, (2)
iv. the unit tangent vector $\hat{\mathbf{T}}$, (2)
v. the curvature κ and the radius of curvature R, (2)
vi. the unit principal normal $\hat{\mathbf{N}}$, (2)
vii. the tangential component of acceleration, (2)
viii. the normal component of acceleration. (2)

(d) A lift ascends 380 metres in 2 minutes travelling from rest to rest. For the first 30 seconds, it travels with uniform acceleration and for the last 20 seconds, it travels with uniform retardation. For the rest of the time, it travels with uniform speed.

i. Find the uniform speed in m/s. (3)
ii. Find the uniform acceleration in m/s^2. (2)
iii. Find the time taken by the lift to ascend the first 200 metres. (4)

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

(a) Suppose that the trajectories of two particles are given by

$$\mathbf{r}_1(t) = t^2\hat{\imath} + (7t-12)\hat{\jmath} + t^2\hat{\mathbf{k}}$$
$$\mathbf{r}_2(t) = (4t-3)\hat{\imath} + t^2\hat{\jmath} + (5t-6)\hat{\mathbf{k}}$$

respectively. Determine whether or not the particles collide. (3)

(b) At time t, the speed v of a particle moving in a straight line is given by

$$v = \frac{(5-t^2)x^2}{10t^2}$$

where x is the distance covered after time t. If at time $t = 1$, the particle has covered a distance $x = 5/3$, find an expression for x in terms of t. (5)

(c) A ball which is thrown vertically upward with speed u reaches a particular height h after a time t_1 on the way up and a time t_2 on the way down. Let g be the acceleration due to gravity. Show that

i. $u = \frac{1}{2}g(t_1 + t_2)$,

ii. $h = \frac{1}{2}gt_1t_2$,

iii. the maximum height reached is

$$\tfrac{1}{8}g(t_1 + t_2)^2.$$

(12)

QUESTION B3 [*20 Marks*]

(a) An object is dropped from a cliff of height H. After it has travelled a distance D, a second object is dropped. Show that at the instant when the first object reaches the bottom of the cliff, the second object is at a distance d above it given by

$$d = 2\sqrt{DH} - D.$$

(14)

(b) A force of 20 N acts in the direction of the positive z-axis on a body of mass 4 kg. The body starts at the origin with initial velocity $\mathbf{v}(0) = \hat{\imath} - \hat{\jmath}$ (where v_0 is in m/s). Find its position vector and its speed at time t. (6)

QUESTION B4 [*20 Marks*]

(a) A projectile is fired with an initial speed of 100 m/s and an angle of elevation 30°. Assuming $g = 10$ m/s^2, find

i. the velocity vector of the projectile at any time t, (3)

ii. the position vector of the projectile at any time t, (3)

iii. the range of the projectile, (3)

iv. The maximum height reached. (3)

(b) At time $t = 0$ a body of unit mass is located at $x = d$ and is travelling with speed u. It is acted upon by a force $\frac{k}{x}$ away from the origin. Show that

$$x = d \exp\left(\frac{u^2 - v^2}{2k}\right),$$

where v is the speed of the particle at any time t. (8)

QUESTION B5 [*20 Marks*]

In polar coordinates (ρ, θ), the position vector of an arbitrary point (x, y) is given by

$$\mathbf{r} = \rho \cos\theta \hat{\mathbf{i}} + \rho \sin\theta \hat{\mathbf{j}}.$$

Find

(a) $\hat{\boldsymbol{\rho}}$ and $\hat{\boldsymbol{\theta}}$, (4)

(b) the position vector $\mathbf{r}$ in terms of $\hat{\boldsymbol{\rho}}$ and $\hat{\boldsymbol{\theta}}$, (4)

(c) $\dot{\hat{\boldsymbol{\rho}}}$ and $\dot{\hat{\boldsymbol{\theta}}}$, the time derivatives of $\hat{\boldsymbol{\rho}}$ and $\hat{\boldsymbol{\theta}}$, (4)

(d) the velocity $\mathbf{v}$ in terms of $\hat{\boldsymbol{\rho}}$ and $\hat{\boldsymbol{\theta}}$, (4)

(e) the acceleration $\mathbf{a}$ in terms of $\hat{\boldsymbol{\rho}}$ and $\hat{\boldsymbol{\theta}}$. (4)

QUESTION B6 [*20 Marks*]

(a) Consider a particle with mass m, velocity vector $\mathbf{v}$ and position vector $\mathbf{r}$. Show that if the particle is moving under a central force, its angular momentum is conserved. (5)

(b) Show that movement under a central force occurs in a plane which is perpendicular to the angular momentum $\mathbf{L}$. (5)

(c) When a 4 kg mass attached to a spring, it is observed to oscillate with a period of 2 seconds. Find the period of oscillation if a 6 kg mass is attached to the spring. (5)

(d) A simple pendulum of length 0.6 m with a block of mass 5 kg has a maximum speed of 1.2 m/s. Find the maximum height reached by the block. (Assume $g = 10$ m/s^2.) (5)

END OF EXAMINATION PAPER