University of Swaziland

Re-Sit/Supplementary Examination, 2017/2018

BASS, B.Ed (Sec.), B.Sc.

Title of Paper : Dynamics I

Course Number : MAT256/M255

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth a total of 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2, B3 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.

Special Requirements: NONE

This examination paper should not be opened until permission has been given by the invigilator.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

(a) Let $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{b} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$. Find a unit vector perpendicular to both $\mathbf{c} = \mathbf{a} - \mathbf{b}$ and $\mathbf{d} = \mathbf{a} + \mathbf{b}$. (5)

(b) Let $\mathbf{F}(x, y, z) = 3x^2y\hat{\mathbf{i}} + (x^3 + y^3)\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$. Show that $\mathbf{F}$ is a conservative vector field and find a scalar potential for $\mathbf{F}$. (10)

(c) A particle moves along the space curve defined by

$$\mathbf{r}(t) = 4\sin t\hat{\mathbf{i}} + 4\cos t\hat{\mathbf{j}} + 8\hat{\mathbf{k}}.$$

Find

i. the velocity of the particle, (2)
ii. the speed of the particle, (2)
iii. the acceleration of the particle, (2)
iv. the unit tangent vector $\hat{\mathbf{T}}$, (2)
v. the curvature κ and the radius of curvature R, (2)
vi. the unit principal normal $\hat{\mathbf{N}}$, (2)
vii. the tangential component of acceleration, (2)
viii. the normal component of acceleration. (2)

(d) A train takes time T to perform a journey. It first travels with uniform acceleration for a time $\frac{T}{n}$ then travels with uniform speed V for a time $(n-2)\frac{T}{n}$ and finally travels under uniform retardation for a time $\frac{T}{n}$.

i. Prove that the average speed of the train is

$$(n-1)\frac{V}{n}.$$

(5)

ii. If the length of the journey is 64 km, the time taken to complete the journey is $T = 50$ minutes, and the uniform speed is $V = 96$ km/h, find the time taken to complete that part of the journey that is under this uniform speed. (4)

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

(a) Two points A and B are at a distance d apart. A particle starts from point A and moves in the direction $\overrightarrow{AB}$ with initial velocity u and uniform acceleration a. At the same instant, a second particle moves from point B in the direction of $\overrightarrow{BA}$ with initial velocity $2u$ and uniform retardation a.

i. Show that the particles collide at time

$$T = \frac{d}{3u}$$

after the beginning of motion. (7)

ii. Show that if the particles collide before the second particle returns to B, then

$$ad < 12u^2.$$

(5)

(b) A ball is thrown vertically upwards with speed 18 m/s. Find the greatest height reached and the time taken to reach this height. (Leave your answer in terms of g.) (8)

QUESTION B3 [*20 Marks*]

(a) At time $t = 0$ a particle of mass 2 kg falling under gravity is positioned at the origin and is travelling vertically downward with speed 10 m/s. Suppose that the resisting force at speed v is $-0.5v$ N. Find the speed $v(t)$ and the distance travelled $z(t)$ at any time t. (12)

(b) The acceleration of a particle moving in a straight line is given by

$$a = \frac{10}{4 + 5\sqrt{v}}$$

(in m/s^2) where v (in m/s) is the speed of the particle at distance x (in metres) from the origin. Suppose the particle started from rest at the origin. Determine the distance covered when the particle has reached a speed of 16 m/s. (8)

QUESTION B4 [*20 Marks*]

(a) A particle is projected with speed V at angle of elevation θ. Show that the horizontal distance $\overline{x}$ travelled is given by

$$\overline{x} = \frac{V^2}{g} \sin 2\theta.$$

(8)

(b) A projectile is fired with an initial speed of 200 m/s and an angle of elevation 60°. Assuming $g = 10$ m/s^2, find

i. the velocity vector of the projectile at any time t, (3)

ii. the position vector of the projectile at any time t, (3)

iii. the range of the projectile, (3)

iv. The maximum height reached. (3)

QUESTION B5 [*20 Marks*]

In cylindrical coordinates (ρ, θ, z), the position vector of an arbitrary point (x, y, z) is given by

$$\mathbf{r} = \rho \cos\theta \hat{\imath} + \rho \sin\theta \hat{\jmath} + z\hat{\mathbf{k}}.$$

Find

(a) $\hat{\boldsymbol{\rho}}$, $\hat{\boldsymbol{\theta}}$ and $\hat{\mathbf{z}}$, (5)

(b) the position vector $\mathbf{r}$ in terms of $\hat{\boldsymbol{\rho}}$, $\hat{\boldsymbol{\theta}}$ and $\hat{\mathbf{z}}$, (3)

(c) $\dot{\hat{\boldsymbol{\rho}}}$, $\dot{\hat{\boldsymbol{\theta}}}$, and $\dot{\hat{\mathbf{z}}}$, the time derivatives of $\hat{\boldsymbol{\rho}}$, $\hat{\boldsymbol{\theta}}$ and $\hat{\mathbf{z}}$, (6)

(d) the velocity $\mathbf{v}$ in terms of $\hat{\boldsymbol{\rho}}$, $\hat{\boldsymbol{\theta}}$ and $\hat{\mathbf{z}}$, (3)

(e) the acceleration $\mathbf{a}$ in terms of $\hat{\boldsymbol{\rho}}$, $\hat{\boldsymbol{\theta}}$ and $\hat{\mathbf{z}}$. (3)

QUESTION B6 [*20 Marks*]

(a) Show that

$$x(t) = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t$$

can be written as

$$x(t) = A \cos\left(\omega_0 t + \phi\right)$$

where $A = \sqrt{c_1^2 + c_2^2}$ and $\phi = \arctan\left(-c_2/c_1\right)$. (6)

(b) Find the amplitude and phase constant for the block-spring system with position given by

$$x(t) = \sqrt{3}\cos \omega_0 t - \sin \omega_0 t.$$

(4)

(c) A block of mass m is attached to a spring with spring constant k and is free to slide along a frictionless surface. At $t = 0$, the system is stretched at an amount $x_0 > 0$ from the equilibrium position and is released from rest.

Find the period of oscillation of the block and find the speed of the block when it first returns to the equilibrium position. (10)

END OF EXAMINATION PAPER