UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION, 2017/2018

BASS III, B.Ed (Sec.) III, B.Sc III, B.Eng IV

Title of Paper : NUMERICAL ANALYSIS I

Course Number : MAT 311/M311

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

A1 (a) Suppose you need to evaluate $f(x) = \sqrt{x^4 + 4} - 2$ for x near 0.

i. Show that a direct calculation of $f(0.5)$ using the definition of $f(x)$ with 3-digit rounding arithmetic can lead to large relative errors. What causes the errors? [5 Marks]

ii. Derive an alternative formula for $f(x)$ that has better round-off error properties. Illustrate by using your formula to calculate $f(0.5)$ and the corresponding relative error. [5 Marks]

(b) Consider the definite integral

$$\int_1^2 x\ lnx\ dx$$

i. Approximate the integral using Trapezoid Rule with $n = 4$. [5 Marks]

ii. Determine bounds on the values of n and h required to approximate the integral to within 10^{-4}. [5 Marks]

(c) Beginning with $x_0 = 1$, write the first three iterations of Newton's method for the equation $x^3 + x = 1$ [5 marks]

(d) The quadrature formula

$$\int_{-1}^{1} f(x)\ dx \approx c_0 f(-1) + c_1 f(0) + c_2 f(1)$$

is exact for all polynomials of degree less than or equal to 2.
Determine c_0, c_1 and c_2. [5 Marks]

(e) i. Convert the decimal 14.9 to its binary equivalent [5 Marks]

ii. Convert the binary $100100.\overline{0110}$ to its decimal equivalent [5 Marks]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

B2 (a) Determine the decimal number that has the following single-precision representation

$$01000011010010001111000000000000$$

[7 Marks]

(b) Determine the machine representation in single precision on a 32-bit word length computer (Marc-32) for the decimal number -200.9375. [7 Marks]

(c) Consider the equation $f(x) = x^3 + 4x^2 - 10 = 0$. Prove that there exists a unique solution of the equation in [1,2] [6 Marks]

QUESTION B3 [*20 Marks*]

B3 (a) The nth root of the number N can be found by solving the equation $x^n - N = 0$. Show that the Newton's method for this equation is

$$x_{i+1} = \frac{1}{n}\left[(n-1)x_i + \frac{N}{x_i^{n-1}}\right]$$

[5 Marks]

(b) Use the result in (a) to approximate $130^{\frac{1}{3}}$ using $x_0 = 5$ as a starting guess. [5 Marks]

(c) State the fixed point theorem [3 Marks]

(d) The equation $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in $[1, 2]$. Prove that the sequence of iterations

$$x_{n+1} = g(x_n), \quad \text{where } g(x) = \sqrt{\frac{10}{4+x}}$$

converges to the solution of the equation $f(x) = 0$ for any initial guess $x_0 \in [1, 2]$.
[7 Marks]

QUESTION B4 [*20 Marks*]

B4 (a) Construct a Newton's forward difference table corresponding to the following data and find a polynomial of least degree that goes through the points.

x	-5	-2	1	4
$f(x)$	25	4	2	23

[10 marks]

(b) Example Given the following data table

x	-2	-1	1	2
$f(x)$	-4	-2	6	12

Derive the Newton interpolation polynomial *directly* and use it to approximate $f(0)$.
[10 marks]

QUESTION B5 [*20 Marks*]

B5 (a) Construct a quadrature rule on the interval $[0, 4]$ using the nodes 1, 2, 3. [10 Marks]

(b) The quadrature formula $\int_{-1}^{1} f(x)\, dx \approx c_0 f(x_0) + c_1 f(x_1)$ has the highest degree of precision. Determine x_0, c_0, c_1 and x_1 and use the resulting formula to approximate the integral

$$\int_{-1}^{1} x^4\, dx$$

[10 Marks]

QUESTION B6 [*20 Marks*]

B6 (a) Find the LU factorisation of the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & 4 & 5 & 1 \\ 8 & 5 & 0 & 3 \end{bmatrix}$$

where L is a lower triangular matrix with ones on the main diagonal and U is an upper triangular matrix. [10 Marks]

(b) Solve the system $A\mathbf{x} = b$ using the LU factorisation obtained in (a) when $b = [1, 1, 2, 0]^T$ [10 Marks]

END OF EXAMINATION PAPER