# UNIVERSITY OF SWAZILAND

## SUPPLEMENTARY EXAMINATION, 2017/2018

## B.Sc. III, B.Eng III, B.Ed III, BASS III

**Title of Paper** : Vector Analysis

**Course Number** : M312/MAT312

**Time Allowed** : Three (3) Hours

**Instructions**

1. This paper consists of TWO (2) Sections:
   a. SECTION A (40 MARKS)
      - Answer **ALL** questions in Section A.
   b. SECTION B
      - There are FIVE (5) questions in Section B.
      - Each question in Section B is worth 20 Marks.
      - Answer **ANY THREE (3)** questions in Section B.
      - If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked.**
2. Show all your working.

**Special Requirements: None**

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

## SECTION A [*40 Marks*]: Answer ALL Questions

A1. Find the equation of the plane which passes through the point $(1, 2, 3)$ and is normal to the vector $-\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + 2\hat{\boldsymbol{k}}$. (5)

A2. Express the vector $\overline{F}(-y, x, 0)$ in cylindrical coordinates. (5)

A3. Find the unit vector normal to the surface $xyz = 2$ at the point $(1, -1, -2)$. (5)

A4. Show that the vector

$$\overline{F} = (x^3 z - 2xyz)\hat{\boldsymbol{i}} + (xy - 3x^2 z)\hat{\boldsymbol{j}} + (yz^2 - xz)\hat{\boldsymbol{k}}$$

is solenoidal. (5)

A5. Find the work done by force $\overline{F} = 3xy\hat{\boldsymbol{i}} - y^2\hat{\boldsymbol{j}}$ along the curve

$$C = \{x, y : y = 2x^2, \text{ from } (0,0) \text{ to } (1,2)\}$$

(5)

A6. (a) Apply the Green's theorem to show that the area bounded by a simple closed curve $C$ is given by $\frac{1}{2}\oint_c xdy - ydx$. (5)

(b) Use line integral to evaluate the area enclosed by the straight lines $y = x, y = -x, y = 4$ (5)

A7. Let $\overline{A} = \nabla f$ and $\nabla^2 f = -4\pi\rho$. Apply the divergence theorem to show that

$$\int\int_S \overline{A} \cdot \overline{n} ds = -4\pi \int\int_V \int \rho \ dv.$$

(5)

## SECTION B: Answer Any *THREE* Questions

### QUESTION B1 [*20 Marks*]

B1. (a) Find the equation of the plane containing the point $(-2, -5, 2)$ and the line

$$\frac{x-7}{1} = \frac{y}{3} = \frac{z-1}{-5}$$

(6)

(b) Find the acute angle between the planes $5x - y - 2z = 5$ and $y = 0$ (5)

(c)

(i) Find the direction along which the directional derivative of $f = x^2 y z^3$ at point $(2, 1, -1)$ is greatest.

(ii) Determine this greatest value.

(iii) Find the rate of change of this $f$ at the same point in the direction of a positive $z$-axis (4,2,3)

## QUESTION B2 [*20 Marks*]

B2. (a) Elliptical curvilinear coordinates $\xi, \eta, \zeta$ are such that the position vector is given by

$$\overline{r} = (\cos h\xi \cos\eta,\ \ \sin h\xi \sin\eta,\ \ \zeta),$$

where $0 \leq \xi < \infty, \pi < \eta \leq \pi, -\infty < \zeta < \infty$.

(i) Show that this system is orthogonal.

(ii) Find the Lame parameters. (6,4)

(b) For the curvilinear orthogonal coordinates system of $q_1, q_2 q_3$ the velocity and the components of acceleration are given by

$$\overline{V} = \sum_{l=1}^{3} H_l \dot{q}_l \bar{e}_l, \qquad a_l = \frac{1}{H_l}\left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_l}\right) - \frac{\partial T}{\partial q_l}\right].$$

in the usual notations.

Pass the spherical coordinates. Derive the formulas for

(i) Velocity,

(ii) Acceleration. (3,7)

## QUESTION B3 [*20 Marks*]

B3. (a)Prove the following formulae

(i) $\nabla \cdot (f\overline{A}) = f\nabla \cdot A + \nabla f \cdot \overline{A}$.

(ii) $(\nabla \times \nabla f) = 0$.

(iii) $\nabla \cdot (\nabla \times \overline{A}) = 0$. (3,2,4)

(b) Consider a vector field

$$\overline{F} = (\sin y + z)\hat{\boldsymbol{\imath}} + (x\cos y - z)\hat{\boldsymbol{\jmath}} + (x - y)\hat{\boldsymbol{k}}.$$

(i) Is $\overline{F}$ irrotational? Explain.

(ii) Is $\overline{F}$ conservative? Explain.

(iii) If so, find a scalar potential. (4,2,5)

## QUESTION B4 [*20 Marks*]

B4. (a) Evaluate

$$\int_C z\, d\overline{r}$$

where $C$ is the curve $\overline{r} = (a\cos t, b\sin t, ct)$ from the point $t = 0$ to the point $t = 2\pi$. (7)

(b) Evaluate

$$\int_C (y - \sin x)dx + \cos x dy,$$

where $C$ is a triangle with the vertices

$(0,0), (\frac{\pi}{2}, 0)$ and $(\frac{\pi}{2}, 1)$

(i) Directly,

(ii) by Green's theorem. (6,7)

**QUESTION B5** [*20 Marks*]

B5. (a) Apply the divergence theorem to show that for any differentiable vector $\overline{A} = (A_x, A_y, A_z)$ it follows

$$\int\int_S (A_x dydz + A_y dzdx + A_z dxdy) = \int\int\int_V \left( \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) dxdydz$$

in the usual notations. (5)

(b) Evaluate $\int\int_S \overline{A} \cdot d\overline{S}$ where

$$\overline{A} = 4xz\hat{\boldsymbol{\imath}} - y^2\hat{\boldsymbol{\jmath}} + yz\hat{\boldsymbol{k}},$$

and $S$ is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$

(i) Directly.

(ii) Apply the divergence theorem to pass to the triple integral. (8,7)

END OF EXAMINATION PAPER