UNIVERSITY OF SWAZILAND
FINAL EXAMINATION, 2017/2018
BASS I

Title of Paper   :   Elementary Quantitative Techniques II

Course Number   :   MAT102

Time Allowed   :   Three (3) Hours

Instructions

1.   This paper consists of TWO (2) Sections:

    a.   SECTION A (40 MARKS)
         —   Answer **ALL** questions in Section A.
    b.   SECTION B
         —   There are FIVE (5) questions in Section B.
         —   Each question in Section B is worth 20 Marks.
         —   Answer **ANY THREE** (3) questions in Section B.
         —   If you answer more than three (3) questions in Section B, **only the first three questions answered in Section B will be marked.**

2.   Show all your working.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION A1

(a) Use the limit definition to find $f'(x)$

(i) $f(x) = 3x^2 + 2x + 5$

(ii) $f(x) = \sqrt{x+1}$ [8 marks]

(b) Find the first derivative and second derivative of the following

$y = x^2 \sin x$ [8 marks]

(c) Find the equation of the tangent to the curve

$y = x^4 - 4x^2$ at the point $(3, 45)$ [4 marks]

QUESTION A2

(a) Evaluate the following integrals

(i) $\int \left(x^4 + x + \dfrac{1}{\sqrt{x}} + \dfrac{1}{x^2} \right) dx$

(ii) $\int \dfrac{5x+5}{(x-1)(x+3)} dx$ [10 marks]

(b) Find the area bounded by the curve $y = x - x^2$ and the x-axis $(y = 0)$.

[10 marks]

QUESTION B3

(a) Find $f'(x)$ for

$$f(x) = x \sin x$$
[5 marks]

(b) Evaluate the following integral using partial fractions

$$\int \frac{3x^2 - 2x + 1}{x^3 - x^2} dx$$
[10 marks]

(c) Evaluate the following integral:

$$\int x\left(x^2 + 3\right)^8 dx$$
[5 marks]

QUESTION B4

(a) Find the area of the region lying above the x-axis and under the parabola
$$y = 4x - x^2$$
[10 marks]

(b) Find all relative maxima and relative minima of the functions
(i) $y = x^3 - 3x + 1$
(ii) $y = x^4 - 18x^2$
[10 marks]

QUESTION B5

(a) The total cost of producing q units of a certain product is
$$C = 200\,000 + 5000q - 0.1q^2$$
where C is the total cost stated in Emalangeni

(i) Determine how many units q should be produced in order to minimise the total cost.

(ii) What is the total cost of production at this level of output?
[10 marks]

(b) Calculate the area bounded by the curves $y = x^2 - 1$ and $y = 1 - x^2$.
[10 marks]

QUESTION B6

(a) Find the equation of the tangent to the curve $y = x^4 - 2x^3 + 3$ at the point $(-1, 6)$. [5 marks]

(b) Find the area of the region enclosed by the curves $y = 10 + 3x - x^2$ and $y = 2x + 4$ [10 marks]

(c) Find the integral

$$\int \sin^3 x \cos^3 x \, dx$$ [10 marks]

QUESTION B7

(a) Find the derivative of y with respect to x for

$$y = x^{2x}$$ [5 marks]

(b) Evaluate the following integral

$$\int \frac{\cos x}{1 + \sin x} dx$$ [5 marks]

(c) Use implicit differentiation to find the derivative of y with respect to x for

$$e^y - e^x = e^{y-x}$$ [10 marks]