

UNIVERSITY OF SWAZILAND
SUPPLEMENTARY EXAMINATION, 2017/2018
BASS I

Title of Paper : Elementary Quantitative Techniques II

Course Number : MAT102

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO (2) Sections:

a. SECTION A (40 MARKS)

– Answer ALL questions in Section A.

b. SECTION B

– There are FIVE (5) questions in Section B.

– Each question in Section B is worth 20 Marks.

– Answer ANY THREE (3) questions in Section B.

– If you answer more than three (3) questions in Section B, only the first three questions answered in Section B will be marked.

2. Show all your working.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A
ANSWER ALL QUESTIONS

QUESTION A1

(a) Use the limit definition to find $f'(x)$

(i) $f(x) = \frac{1}{x}$

(ii) $f(x) = \frac{x+1}{x+2}$

[8 marks]

(b) Use implicit differentiation to find $f'(x)$

$$y^2 + y + x^2 + x = 0$$

[6 marks]

(c) Find the derivative of y with respect to x if

$$y = x^{\sin x}$$

[6 marks]

QUESTION A2

(a) Evaluate the following integrals

(i) $\int (x^2 - 4x + 5)^5 (x - 2) dx$

(ii) $\int \frac{e^{1/x}}{x^2} dx$

[10 marks]

(b) Evaluate the following by means of partial fractions:

$$\int \frac{dx}{x^3 - 2x^2 + x}$$

[10 marks]

SECTION B
ANSWER ANY THREE QUESTIONS

QUESTION B3

- (a) Consider the function $f(x) = x^3 - 3x^2 - 24x + 7$

Determine the open intervals over which $f(x)$ is increasing or decreasing.

[10 marks]

- (b) Determine the value of the constant b given that the tangent to the curve

$$y = x^2 + bx + 1$$

at the point $(0, 1)$ is parallel to the line $y = 2x - 4$

[10 marks]

QUESTION B4

- (a) Differentiate and simplify

$$f(x) = x^2 \cos x - 2x \sin x$$

[8 marks]

- (b) Evaluate the integrals

(i) $\int x \sin x \, dx$

(ii) $\int x(x^2 + 3)^8 \, dx$

[12 marks]

QUESTION B5

- (a) The demand function for a certain product is

$$p(x) = 600 - 0.015x$$

where $p(x)$ is the price per unit, in Emalangen, and x is the quantity demanded.

For what value of x is the revenue a maximum? What is the maximum possible revenue? What is the selling price per unit that maximise revenue?

[10 marks]

- (b) Calculate the area of the finite region bounded by the curve $y = x^2 + 2$ and

the straight line $y = 3x$

[10 marks]

QUESTION B6

- (a) When a manufacturing firm sells x unit of its products per month, it sets the unit price at $S(x)$ where

$$S(x) = 100 - \frac{x^2}{10000}$$

The total cost $C(x)$ of producing the x units per month is given by

$$C(x) = \frac{3}{200}x^2 + 40x + 10$$

Determine how many units the company should sell per month in order to realise maximum profits. [12 marks]

- (b) Evaluate the definite integral

$$\int_0^5 \frac{x dx}{x^2 + 9}$$

[8 marks]

QUESTION B7

- (a) Use the definition to find the derivative of the following functions:

(i) $x^2 + x$

(ii) $\frac{1}{x^2}$

[10 marks]

- (b) Evaluate the following integrals:

(i) $\int x\sqrt{1+x^2} dx$

(ii) $\int \sin^5 x \cos^3 x dx$

[10 marks]