UNIVERSITY OF ESWATINI

MAIN EXAMINATION, 2018/2019

BASS I, B.Ed I, B.Comm I

Title of Paper : Algebra, Trigonometry and Analytic Geometry

Course Number : MAT 107/MAT 121/MS 101

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate whether you are full time or part time student and indicate your program on your answer booklet.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

a) Find the number of positive and negative real zeros possible for the polynomial [5]

$$P(x) = x^4 - 6x^3 + 8x^2 + 2x - 1.$$

b) Express [5]

$$3\log_a(x^2) + 6\log_a(x-1) - \log_a(x)$$

as a single logarithm with a coefficient of 1.

c) Prove that $(1 + \tan x)(1 + \cot x) = 2 + \sec x \csc x$. [5]

d) Write and simplify the first three terms in the expansion of $(x - 2y)^6$. [5]

e) Using the method of mathematical induction, prove that

$$2 + 4 + 6 + \cdots + 2n = n(n+1)$$

for all positive integers. [5]

f) The sum of n terms of an arithmetic progression $-2, 2, 6, \cdots$ is 160. Find n. [5]

g) Solve the following linear system of equations using Crammer's rule. [5]

$$2x + 3y = 11$$
$$7x - 2y = 1$$

h) Find the center and radius of a circle given by the equation $x^2 + y^2 - 2y = 48$. [5]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

a) Solve the following linear system of equations using Crammer's rule. [12]

$$\begin{aligned} 2x + 4y + 6z &= 18 \\ 4x + 5y + 6z &= 24 \\ 3x + y - 2z &= 4 \end{aligned}$$

b) Prove by mathematical induction that the following formula [8]

$$1 + 3 + 6 + \cdots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$$

is valid for all positive integers.

QUESTION B3 [*20 Marks*]

a) Find all the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$. [12]

b) Find a value of k so that when $x^2 + 4x + 7$ is divided by $x + k$, the remainder is 3. [8]

QUESTION B4 [*20 Marks*]

a) What amount must Amanda invest at 12% compounded annually, to accumulate $E3000.00$ at the end of 10 years? [12]

b) Solve the logarithmic equation [8]

$$2\log_7(x+1) + \log_7(x-5)^2 = 2.$$

QUESTION B5 [*20 Marks*]

a) Find the coefficient of the term involving x^8 in the expansion of $\left(x^2 - \frac{1}{x}\right)^7$. [8]

b) i) Find the twenty first term of an arithmetic progression whose $9th$ term is 16 and $40th$ term is 47. [6]

ii) The first term of a geometric progression is 4 and the common ratio is 2. Find the sum of the first ten terms. [6]

QUESTION B6 [*20 Marks*]

a) Solve the equation $z^2 + 4z + 5 = 0$ and express your answer in the form $z = a \pm ib$. [5]

b) Given that the line $y + 2x = p$ and the line $3y + \alpha x = 5$ are parallel, find the value of α. [5]

c) Find the equation of the circle with the center at $(4, -3)$ and radius of $r = 7$. [5]

d) If $x = 2\cos(\theta)$ and $y = 3\sin(\theta)$, prove that $9x^2 + 4y^2 = 36$. [5]

END OF EXAMINATION PAPER