UNIVERSITY OF ESWATINI

RESIT/SUPPLEMENTARY EXAMINATION, 2018/2019

BASS I, B.Ed I, B.Comm I

Title of Paper : Algebra, Trigonometry and Analytic Geometry

Course Number : MAT 107/MAT 121/MS 101

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate whether you are full time or part time student and indicate your program on your answer booklet.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

a) Find the number of positive and negative real zeros possible for the polynomial [5]

$$P(x) = -x^4 + 6x^3 - 8x^2 + 2x - 1.$$

b) Express [5]

$$\log_4(x^2 - 1) - \log_4(x - 1) + 3\log_4(x)$$

as a single logarithm with a coefficient of 1.

c) Prove that $(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$. [5]

d) Write and simplify the first two terms in the expansion of $(\frac{2}{x} - y)^5$. [5]

e) Using the method of mathematical induction, prove that

$$4 + 8 + 12 + \cdots + 4n = 2n(n + 1)$$

for all positive integers. [5]

f) Find the tenth term of the arithmetic progression $-2, 2, 6, \cdots$. [5]

g) Solve the following linear system of equations using Crammer's rule. [5]

$$2p + 3q = 11$$
$$-2q + 7p = 1$$

h) Find the equation of the line which is perpendicular to $3x - 2y - 4 = 0$ and passes through the point $(0, 2)$ [5]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

a) Solve the equation $5z^2 + 2z + 10 = 0$ and simplify your answer. [6]

b) Find the equation of the line that is perpendicular to the line $y = 2 - \frac{1}{2}x$ and passing through the point $(1, -1)$. [6]

c) Show that $\frac{1}{1+\sin x} + \frac{1}{1-\sin x} = 2\sec^2(x)$ [8]

QUESTION B3 [*20 Marks*]

a) Find all the real roots of the equation $2x^3 - 12x^2 + 22x - 12 = 0$. [12]

b) i) Determine whether $x + 3$ is a factor of $6x^3 + 19x^2 + 2x - 3$ [4]

ii) Find the remainder when $x^5 - 1$ is divided by $x - 2$. [4]

QUESTION B4 [*20 Marks*]

a) How many years will be needed for $E5000.00$ to increase to $E18000.00$ at 12% compounded weekly? [12]

b) Solve the exponential equation [8]

$$3^{2x+1} - 5^{x+1} = 0.$$

QUESTION B5 [*20 Marks*]

a) Find the coefficient of the term involving x^8 in the expansion of $\left(x^2 - \frac{1}{x}\right)^7$. [8]

b) i) Find the twenty first term of an arithmetic progression whose $9th$ term is 16 and $40th$ term is 47. [6]

ii) The first term of a geometric progression is 4 and the common ratio is 2. Find the sum of the first ten terms. [6]

QUESTION B6 [*20 Marks*]

a) Solve the following linear system of equations using Crammer's rule. [12]

$$\begin{aligned} x + 2y + z &= 1 \\ x - y - z &= 0 \\ 2x + y + z &= 3 \end{aligned}$$

b) Prove by mathematical induction that the following formula [8]

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

is valid for all positive integers.

END OF EXAMINATION PAPER