

UNIVERSITY OF ESWATINI

Final Examination, May 2019

B.A.S.S., B.Comm, B. Ed, D.Comm(IDE)

Title of Paper : Calculus for Business Studies

Course Code : MAT108/MS102

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
 - a. **SECTION A (COMPULSORY): 40 MARKS**
Answer ALL QUESTIONS.
 - b. **SECTION B: 60 MARKS**
Answer ANY THREE questions.
Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Special requirements: None

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [40 Marks]: ANSWER ALL QUESTIONS

QUESTION A1 [40 Marks]

a. Evaluate the limits:

$$(a) \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} \quad [3]$$

$$(b) \lim_{x \rightarrow 0} \frac{x^3 - 2x^2}{x^4 + 3x^2} \quad [3]$$

$$(c) \lim_{x \rightarrow \infty} \frac{2x + 5}{1 - 12x} \quad [2]$$

b. Differentiate;

$$(a) y = (x^2 + 1)e^{x^2+1} \quad [3]$$

$$(b) y = \ln(x^2 - 3x + 2) \quad [2]$$

$$(c) y = \sin(4x^2 + 1) \quad [2]$$

c. Find and classify all critical points of $f(x) = x^4 - 2x^2 + 2$. [6]

d. Find the equation of the tangent line to $y = 2x^3 + 3x + 1$ at $(1, 6)$. [4]

e. Find the equation of the curve that passes through $(2, 3)$ if its slope is given by $\frac{dy}{dx} = 4x - 3$ for each x . [3]

f. Evaluate;

$$(a) \int \left(2e^x + \frac{3}{x} \right) dx \quad [2]$$

$$(b) \int \left(\frac{3x^2}{x^3 + 1} \right) dx \quad [3]$$

$$(c) \int \cos(8x) dx \quad [2]$$

g. Find the area of the region bounded by $y = x^2$, $x = 4$ and the x -axis.

[5]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [20 Marks]

a) Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$. [10]

b) Consider the function $f(x) = \frac{x^2 - 1}{x + 1}$.

i) Find and classify the point of discontinuity. [3]

ii) Define $f(x)$ at the point of discontinuity such that $f(x)$ is extended to be continuous at the point of discontinuity. [7]

QUESTION B3 [20 Marks]

a) Use the limit definition of the derivative to find $f'(x)$ given that $f(x) = \frac{1}{x^2}$. [10]

b) A manufacturer of note pads has determined that the cost of producing x note pads each week is given by $C(x) = 40 + 5x + \frac{x^2}{4}$.

i) Find the exact cost of producing the 21st note pad. [2]

ii) Find the marginal cost of producing the 21st note pad and show how the marginal cost closely approximates the cost in (i.) above. [8]

QUESTION B4 [20 Marks]

- a) A computer firm is marketing a new computer model. It determines that in order to sell x computers, the price per computer must be $p = 280 - 0.4x$.
It also determines that the total cost of producing x computers is given by $C(x) = 500 + 0.6x^2$.
What price per computer must be charged in order to make maximum profit? [8]
- b) Consider the function $y = x^3 - 6x^2$.
- i) Find all critical points of y . [2]
 - ii) Use the second derivative test to classify the critical points found in i) above. [4]
 - iii) Find all values of x on which the curve is increasing and/or decreasing. [6]

QUESTION B5 [20 Marks]

Integrate;

- a) $\int x \sin(x^2 + 1) dx$. [5]
- b) $\int (x + 1)e^x dx$. [5]
- c) $\int \frac{x - 1}{(2x + 1)(3x + 1)} dx$. [7]
- d) $\int_{-1}^2 (3x^2 + 3) dx$. [3]

QUESTION B6 [20 Marks]

a) Calculate the change in total profits if a company with marginal revenue and marginal cost is given by $R'(x) = \frac{2400}{(10+x)^2}$ and $C'(x) = 0.02x + 5$ increases production from $x = 20$ to 30 units. [6]

b) Consider the price-demand and price-supply equations;

$$p = S(x) = 5x^2 + 60x, \quad p = D(x) = 200 - 15x^2.$$

i) Find the equilibrium quantity x^* and the equilibrium price p^* . [4]

ii) Find the consumer's surplus at the equilibrium price level. [5]

iii) Find the producer's surplus at the equilibrium price level. [5]

END OF EXAMINATION