# University of Swaziland

## Final Examination – December 2018

**BSc I, BEng I, BEd I, BASS I, BSc IT I, BSc Comp Sci Ed I**

**Title of Paper** : Algebra, Trigonometry and Analytic Geometry
**Course Number** : MAT111
**Time Allowed** : Three (3) hours

**Instructions:**

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 3 (out of 5) questions in Section B.
4. Show all your working.
5. Begin each question on a new page.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

## Section A

**Answer ALL Questions in this section**

**A.1** a. Given that

$$\frac{P(x)}{D(x)} = Q(x) \text{ rem } R(x),$$

where $P$, $D$, $Q$ and $R$ are all polynomials,

i. state the *inequality* satified by the degrees of $P$, $D$ and $R$ [1 mark]

ii. state the *equation* satified by the degrees of $P$, $D$ and $Q$ [1 mark]

b. On the same coordinate axes, sketch the graphs of $y = e^x$, $y = \ln x$ and $x^2 + y^2 = 4$. [3 marks]

c. Without using a calculator, evaluate

i. $2\log_6 3 + \log_6 20 - \log_6 5$ [4 marks]

ii. $\left(\cos 255^0 - \sin 255^0\right)^2$ [4 marks]

d. Using a calculator, find the value of (correct to 2 d.p.)

$$4e^{2\pi} + \frac{\log_2 35 + \ln 20}{1 - \frac{\log 20}{2 - \log 40}}.$$ [2 marks]

e. Find the value of

i. $\sum_{n=0}^{200}(7n + 9)$ [4 marks]

ii. $5 + 10 + 20 + \cdots + 1,310,720$ [4 marks]

f. Find the *8th term* in the binomial expansion of

$$\left(x^2 + \frac{1}{x}\right)^{24}.$$ [5 marks]

g. Given that $i$ is the unit imaginary number, satisfying the property $i^2 = -1$, evaluate

$$\begin{vmatrix} 1 & 0 & i \\ -2i & 7i & 1 \\ 1 & 4 & -9i \end{vmatrix}.$$ [6 marks]

h. Use *synthetic division* to find the quotient and remainder of

$$\frac{x^4 - 2x^2 + 3x - 9}{x + 2}.$$ [6 marks]

## Section B

**Answer ANY 3 Questions in this section**

**B.1** a. Given the vectors $A = 2\hat{i} + 5\hat{j} - 8\hat{k}$ and $B = 4\hat{i} - 5\hat{k}$, find

i. $A \cdot B$ [2 marks]

ii. $A \times B$ [4 marks]

iii. the angle between the vectors $A$ and $B$ (correct to 2 d.p.) [4 marks]

b.

i. Find the quotient and remainder of

$$\frac{5x - 5x^3 + x^4 - 9}{x^2 - 3}.$$ [5 marks]

ii. Using the Rational Root Theorem, the Remainder Theorem and synthetic division, factorise

$$P(x) = 2x^3 + 3x^2 - 18x + 8.$$ [5 marks]

**B.2** a. Express

$$2\ln\left(ab^2c^3\right) - \ln\left(a^2b^3c^6\right)$$

as a single logarithm with unit coefficient. [3 marks]

b. Consider the function

$$y(t) = \frac{500e^{0.2t}}{4 + e^{0.2t}}.$$

i. Find the value of $y(20)$ correct to 2 d.p. [2 marks]

ii. Make $t$ the subject of the formula. [6 marks]

iii. Hence, or otherwise, find the value of $t$ (correct to 2 d.p.) when $y = 350$. [2 marks]

c. A parent opens a saving account for their child by making monthly deposits, beginning on 31 January 2018. The following table shows the first few deposits.

| *Month* | Jan '18 | Feb '18 | March '18 | April '18 | May '18 |
|---|---|---|---|---|---|
| *Deposit (E)* | 200 | 210 | 220 | 230 | 240 |

If the deposits follow the trend shown above for 10 years, find

i. the monthly deposit on 31 December 2021 [2 marks]

ii. the month when the montly deposit will be E1,000 [2 marks]

iii. the *total* deposited in 10 years [3 marks]

**B.3** a. Consider the binomial expansion of

$$\left(x^3 - \frac{y^4}{x^2}\right)^{20}.$$

Find

i. the first 4 terms [6 marks]

ii. the term involving $x^{-20}$. [4 marks]

b. Use mathematical induction to prove the formula

$$\sum_{i=1}^{n}(7i-3) = \tfrac{1}{2}n(7n+1). \quad \text{[10 marks]}$$

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**B.4** a. Without using a calculator, find the *exact* value of

$$\cos 214^0 \cos 334^0 + \sin 214^0 \sin 334^0. \quad \text{[3 marks]}$$

b. Given that $\sin A = \frac{1}{2}$ while $\cos A < 0$, find the *exact* values of

i. $\cos A$ [3 marks]

ii. $\cos 2A$ [4 marks]

c. Prove

$$1 - \frac{\sin^2 A}{1+\cos A} = \cos A. \quad \text{[5 marks]}$$

d. Find the general solution (in radians) of

$$2\sin^2\theta + \cos\theta - 1 = 0. \quad \text{[5 marks]}$$

**B.5** a. Evaluate

$$5 - 3i + \frac{10}{1 - 2i}.$$ [4 marks]

b. Consider the complex number

$$\Omega = 2i\sqrt{3} - 2.$$

i. Express $\Omega$ in polar form [3 marks]

ii. Use de Moivre's theorem to find $\Omega^8$, leaving your answer in the form $a + ib$. [3 marks]

c. A circle is centred on the straight line $2x + 5y + 1 = 0$, and passes through the points $(2, 3)$ and $(-1, 6)$.

i. Find the equation of the circle. [7 marks]

ii. Find the equation of the tangent of the circle at $(-1, 6)$. [3 marks]

END OF EXAMINATION