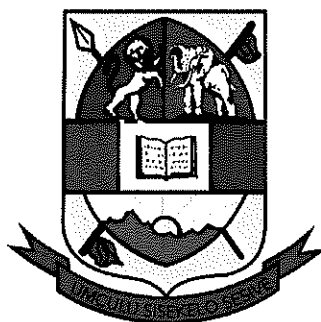# UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION, 2018/2019

Title of Paper : ALGEBRA, TRIG. AND ANALYTIC GEOMETRY

Course Number : MAT111

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.

2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.

3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.

4. Show all your working.

5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.

6. You can answer questions in any order.

7. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

> ## SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

A1 (a) Solve for the value of x in

$$\left(\frac{3}{2}\right)^{2x-1} = \frac{27}{8}.$$

[5 marks]

  (b) A sum of money M_0 Emalangeni is invested in an account that pays interest at $r\%$ p.a. cpmpounded daily. After t years, the amount in the account is given by

$$A(t) = M_0 \left(1 + \frac{r}{365}\right)^{365t}.$$

  i. If the amount of the account doubles in 5 years, find r.   [3 marks]
  ii. Find the time it takes for the amount on the account to tripple.   [2 marks]

A2 (a) Given the following matrices

$$X = \begin{pmatrix} 1 & 3 \\ -1 & 0 \\ 4 & -2 \end{pmatrix}, \quad Y = \begin{pmatrix} 3 & -2 & 4 \\ 2 & 1 & 5 \end{pmatrix} \quad \text{and} \quad Z = \begin{pmatrix} 20 & 101 & 31 \\ 0 & -1 & 1 \\ 1 & 1 & 10 \end{pmatrix}.$$

  Compute (where possible)

  i. $X + 2Y^T$ [2 marks]
  ii. $X - Y$ [2 marks]
  iii. $|Z|$.   [4 marks]

  (b) Given the vectors $\boldsymbol{A} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\boldsymbol{B} = 7\hat{i} - 3\hat{k}$. Find

  i. $\boldsymbol{A} \cdot \boldsymbol{B}$ [2 marks]
  ii. the angle between $\boldsymbol{A}$ and $\boldsymbol{B}$ using the cross product. [5 marks]

A3 (a) Prove the following identity

$$\sec \alpha - \cos \alpha = \sin \alpha \tan \alpha.$$

[5 marks]

  (b) Find the equation of a straight line passing through the points
  $(-4, 8)$ and $(3, -6)$.   [5 marks]

A4 Prove, using mathematical induction, that if i is a complex number satsifying $i^2 = -1$, then for any integer n,

$$i^{4n+1} = i.$$

[5 marks]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

B2 (a) State the *rational root theorem* . [2 marks]

   (b) The remainder when $p(x) = 5x^3 + \beta x^2 + 2x^4 + 3$ is devided by $(x - 2)$ is 35.

      i. Determine the value of β. [2 marks]

      ii. Factorise $P(x)$ and detemine all its roots. [8 marks]

   (c) Consider an arithmetic progression (AP) whose third term is $\frac{25}{4}$ and the seventh term is $\frac{4}{25}$.

      i. Find the first two terms. [2 marks]

      ii. What is the sum of the fisrt 5 terms? [3 marks]

      iii. What is the sum of the 10th and 20th terms? [3 marks]

QUESTION B3 [*20 Marks*]

B3 (a) Find the 5th term of the binomial expansion

$$\left(\frac{1}{x^2} + 4x^2\right)^{\frac{7}{2}}.$$

[5 marks]

   (b) Use synthetic division to determine the quotient and remainder of

$$\frac{2x^4 - 4x + 2x^2 - 9}{2x - 2}.$$

[5 marks]

   (c) On the same axes, sketch the following graphs

$$(i)\ y = \left(\frac{1}{2}\right)^x \quad \text{and} \quad (ii)\ y = \log_{\frac{1}{2}} x.$$

[6 marks]

   (d) make x the subject in the formula $t - \ln x = y - \ln(ax - d)$. [4 marks]

QUESTION B4 [*20 Marks*]

B4 (a) Use Crammer's rule to solve the system

$$x + y + z = 6$$
$$3x - 2y + z = 2$$
$$x + 2y + 3z = 14.$$

[15 marks]

(b) Evaluate the following and express the terms in the form $a + bi$

i.
$$(-4 + 3i) \left(\frac{2}{3} + 2i \right)$$

[2 marks]

ii.
$$\frac{2 + 3i}{-1 + i}$$

[3 marks]

QUESTION B5 [*20 Marks*]

B5 (a) Fid the equation of a parabola with focus $F(-4, 0)$ and directrix $x = 2$ [6 marks]

(b) Prove the identity
$$\frac{1 + \cos A}{1 - \cos A} - \frac{1 - \cos A}{1 + \cos A} = 4 \cot A \csc A.$$

[7 marks]

(c) Given that $\sin x = 2/3$ and $\cos x$ is negative. Find

  i. $\sin 2x.$ [3 marks]

  ii. $\tan 2x.$ [4 marks]

QUESTION B6 [*20 Marks*]

B6 (a) Calculate the value of the sum
$$\sum_{i=0}^{\infty} 50 \left(\frac{4}{9} \right)^i.$$

[3 marks]

(b) FInd the area of a triangle whose vertices are given by
$X(6, -2, 2)$, $Y(10, 8, -3)$ and $Z(-7, 5, 6)$.   [7 marks]

(c) Use mathematical induction to prove that for all integers $n \geq 1$,
$$\sum_{i=1}^{n} 2^i = 2^{n+1} - 2.$$

[10 marks]

END OF EXAMINATION