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UNIVERSITY OF ESWATINI



MAIN EXAMINATION, 2018/2019

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**BSc 1, BEng 1, BEd 1, BASS 1, BSc IT 1, BSc Comp Sci Ed 1**

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**Title of Paper** : INTRODUCTION TO CALCULUS

**Course Number** : MAT112

**Time Allowed** : Three (3) Hours

**Instructions**

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

**Special Requirements: NONE**

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

**SECTION A [40 Marks]: ANSWER ALL QUESTIONS**

- A1 (a) A function  $f(x)$  is said to be continuous at a number  $a$  if  $\lim_{x \rightarrow a} f(x) = f(a)$ . Show that the function  $f(x) = 1 - \sqrt{1 - x^2}$  is continuous at  $x = 1$ . [5 marks]
- (b) i. On the same axes, sketch the graphs  $y = x^2 - 4$  and  $y = 8 - 2x^2$ . [4 marks]  
ii. Determine the area of the region bounded by the two curves in b(i). [4 marks]
- A2 (a) An object moves in a straight line such that its position (in metres) is given by

$$S(t) = 27 - 12t^2 + 2t^3$$

where  $t$  is the time in seconds.

- i. What is the velocity of the object after 2 seconds? [3 marks]  
ii. When is the object instantaneously at rest? [2 marks]
- (b) Differentiate the following functions
- i.  $y = -x + x \ln 2x$   
ii.  $y = \tan^{-1} 2x$

[3,4 marks]

- A3 (a) Evaluate the following limits

i.

$$\lim_{x \rightarrow \infty} \left( \frac{4 + 2x - 9x^2}{6x^2 - x - 1} \right)$$

ii.

$$\lim_{x \rightarrow 0} \left( \frac{\sin^2 x}{x} \right)$$

[2,3 marks]

- (b) Evaluate the following definite integrals

i.

$$\int_{-1}^1 (x^2 - 1) dx$$

ii.

$$\int_1^4 \left( \frac{-7}{\sqrt{x}} + 4x \right) dx$$

iii.

$$\int_0^{\frac{\pi}{2}} \left( \cos x + e^{\frac{2}{\pi}x} \right) dx$$

[2,3,5 marks]

**SECTION B: ANSWER ANY *THREE* QUESTIONS**

**QUESTION B2 [20 Marks]**

B2 (a) Given that

$$f(x) = \frac{1}{\sqrt{x}}$$

Find  $f'(x)$  using the limit definition of derivatives (i.e. From first principles).

[10 marks]

(b) Find  $y''$  if

$$y = \frac{x}{\sqrt{x-1}}$$

Express your answer as a single fraction.

[10 marks]

**QUESTION B3 [20 Marks]**

B3 (a) Sketch the following functions separately

i.  $y = H(x + 4)$

ii.  $y = H(x - 1)$

iii.  $y - 2 = H(x + 4) + H(x - 1)$

where  $H(x)$  denotes the Heaviside function.

[12 marks]

(b) Evaluate the following limits

i.

$$\lim_{x \rightarrow 0} \left( \frac{\sec x - 1}{x} \right)$$

ii.  $\lim_{x \rightarrow 1} h(x)$ , where

$$h(x) = \begin{cases} 7 - 4x, & x < 1 \\ x^2 + 2, & x \geq 1 \end{cases}$$

[5,3 marks]

**QUESTION B4 [20 Marks]**

B4 (a) Work out

$$\int \ln(2x + 3) dx$$

[10 marks]

(b) Given that  $y = x^3 \ln x$ . Show that

$$\frac{d^4 y}{dx^4} = \frac{3!}{x}$$

[10 marks]

**QUESTION B5 [20 Marks]**

B5 (a) Evaluate

i.

$$\int \frac{x dx}{\sqrt{9x^2 - 4}}$$

ii.

$$\int \frac{dx}{x^2 - 6x + 10}$$

[6,9 marks]

(b) Evaluate

$$\lim_{n \rightarrow \infty} \left( \frac{2n}{\sqrt{n^2 + 1}} \right)$$

[5 marks]

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**QUESTION B6 [20 Marks]**

B6 (a) A company needs to design a closed rectangular box with a square base and a capacity of 12000 cm<sup>3</sup>. If the base is to be made out of a heavy-duty material which costs twice as much as the material for the sides and the top, find the dimensions of such a box that will cost the least. [8 marks]

(b) Evaluate the integral

$$\int \frac{3x - 10}{x^2 - 4x + 4} dx$$

[12 marks]

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**END OF EXAMINATION**