
University of Eswatini



Resit Examination – July 2019

BSc I, BASS I, BEng I, BEd I, BSc IT I, BSc Comp Sci Ed I

Title of Paper : Introduction to Calculus

Course Number : MAT112

Time Allowed : Three (3) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 3 questions in Section B.
4. Show all your working.
5. Begin each question on a new page.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN
BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 a. Give a concise definition of each term

i. Indeterminate form [2 marks]

ii. Inflexion point [2 marks]

b. Evaluate

i. $\lim_{x \rightarrow 2} \frac{3x^2 - 11x - 4}{2x^2 - 5x - 12}$ [4 marks]

ii. $\lim_{x \rightarrow \infty} \frac{3x^2 - x - 10}{2x^2 + 3x - 14}$ [4 marks]

c. Find $\frac{df}{dx}$ using the *limit definition* for $f(x) = 7 - 4x$. [4 marks]

d. Find y' given

i. $y = \ln(\sec x + \tan x)$ [4 marks]

ii. $y = \frac{x^2}{x^2 - e^{-2x}}$ [4 marks]

iii. $9x^2 - 12xy + 4y^2 = 10$, where $y = y(x)$. [4 marks]

e. Evaluate

i. $\int \left(\sinh 2x - \frac{8}{x^3} + \frac{8}{x} - \frac{8}{e^{2x}} \right) dx$ [4 marks]

ii. $\int 8xe^{-2x} dx$ [4 marks]

iii. $\int 8xe^{-2x^2} dx$ [4 marks]

Section B**Answer ANY Three (3) Questions in this section**

B.2 a. Evaluate

i. $\lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x}$ [5 marks]

ii. $\lim_{x \rightarrow \infty} \left(\frac{x^2}{x+1} - \frac{x^2}{x-1} \right)$ [5 marks]

b. Use the *limit definition* to find $\frac{df}{dx}$ for

$$f(x) = \frac{1}{1-x}$$
 [10 marks].

B.3 a. Find y' given that

$$y = \ln \left(\frac{e^x}{1 - e^x} \right).$$
 [5 marks]

b. Use Leibnitz's rule to evaluate

$$\frac{d^4}{dx^4} \left[(x^3 - 2x)e^{-2x} \right].$$
 [6 marks]

c. Derive the formula

$$\frac{d}{dx} [\sin^{-1} u(x)] = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}.$$
 [4 marks]

Hence, or otherwise, find y' for

$$y = \sin^{-1}(2x - 1)$$

and simplify.

[5 marks]

B.4 a. Consider the function

$$y = 10 + 8x^2 - x^4.$$

- i. Find the equation of the tangent line at $x = -1$. [4 marks]
 - ii. Find the stationary point(s) of y and determine the nature of each. [4 marks]
 - iii. Find the inflexion point(s) of y [3 marks]
 - iv. Make a sketch of the graph of y . [4 marks]
- b. A rectangle has two of its vertices on the x -axis and the other two on the parabola $y = 27 - 3x^2$. Find the dimensions of such a rectangle with the largest area. [5 marks]
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B.5 Evaluate

- a. $\int \tan^{-1} x dx$ [7 marks]
 - b. $\int_0^{\frac{\pi}{4}} \cos \theta \sin^3 \theta d\theta$ [5 marks]
 - c. $\int_2^5 \frac{dx}{x^2 - 4x + 13}$ [8 marks]
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- B.6** a. Find the *exact* area of the region bounded by the parabola $y = 2x^2 - 8$ and the straight line $x + y + 2 = 0$. [10 marks]
- b. Use calculus methods to derive the formula $A = \pi r^2$ for the area of the circle $x^2 + y^2 = r^2$. [10 marks]

END OF EXAMINATION
