University of ESwatini

Final Examination, December 2018

B.A.S.S. , B.Sc, B.Eng, B.Ed

Title of Paper : Calculus I

Course Number : M211/MAT211

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Begin each major question (A1, B2, etc) on a new page.
3. Each question in Section B is worth 20%.
4. Show all your working.
5. Non programmable calculators may be used (unless otherwise stated).
6. Special requirements: None.
7. Indicate your program next to your student ID number.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A: Answer All Questions

A1.

(a) i. State the Second Derivative Test for local extrema. [4]

ii. Find all critical points of the function, $f(x) = x^3 - 3x + 3$. [3]

iii. Use the Second Derivative Test for local extrema to classify the critical points in (ii) above. [3]

iv. Use L'Hôpital's Rule to show that $\lim_{x \to \infty} \frac{\ln x}{x^2} = 0$. [3]

(b) i. Set up the integral for finding the area of the region bounded by the graphs of $x = 3 - y^2$ and $x = y + 1$. DO NOT EVALUATE. [4]

ii. What is a solid of revolution? [2]

iii. Give two methods for finding the volume of solids of revolution. [2]

(c) i. List the first five terms of the sequence $a_n = (-1)^{n+1}\left(\frac{2}{n}\right)$. [5]

ii. Is the sequence $\left(\frac{-2}{3}\right)^n$ monotonic? Justify. [3]

iii. Define a geometric series, state when it converges, and give the formula for the sum of a convergent geometric series. [5]

iv. State the n-th Term Test for series Divergence. [3]

v. If $0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ diverges. True or False?

If it is false, explain why or give an example that shows it is false. [3]

Section B: Answer Three(3) Questions Only

B2.

Consider the function $f(x) = \dfrac{x^2 - 2x + 4}{x - 2}$.

(a) Identify the domain of $f(x)$. [1]

(b) Find and **classify** all critical points of f. [4]

(c) Find intervals where f is increasing and where it is decreasing. [4]

(d) Find possible points of inflection, if any occur and determine concavity of the graph. [3]

(e) Identify any asymptotes that may exist. [3]

(f) Sketch the graph of f labelling all major points found above including intercepts if any occur. [5]

B3.

(a) Find the area of the region bounded by the graphs of $y = -x^2 + 3x$ and $y = 2x^3 - x^2 - 5x$. [10]

(b) Use the **Shell Method** to find the volume of the solid generated by revolving the plane region between $y = 2x - x^2$ and $y = 0$ about the line $x = 4$. [10]

B4.

(a) Find the arc length of the graph of $y = \dfrac{x^5}{10} + \dfrac{1}{6x^3}$ over the interval $[2,\ 5]$. [10]

(b) Find the area of the surface generated by revolving the curve, $y = \sqrt{2x - x^2},\ \ 0.5 \leq x \leq 1.5$ about the $x-$axis. [10]

B5.

(a) Simplify the ratio of factorials, $\dfrac{(2n+2)!}{(2n)!}$. [3]

(b) Determine the convergence or divergence of the sequence with the given nth term. If the sequence converges, find its limit.

i. $a_n = (-1)^n \left(\dfrac{n}{n+1}\right)$. [3]

ii. $a_n = \dfrac{10n^2 + 3n + 7}{2n^2 - 6}$. [4]

iii. $a_n = \left(1 + \dfrac{7}{n}\right)^n$. [7]

(c) Write an expression for the nth term of the sequence $\dfrac{1}{9}, \dfrac{2}{12}, \dfrac{2^2}{15}, \dfrac{2^3}{18}, \dfrac{2^4}{21}, \ldots$. [3]

B6.

(a) i. Show that the infinite series converges, $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$. [5]

ii. Use the Ratio Test to determine the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{3^n}$. [5]

iii. Use Direct Comparison Test to determine the convergence or divergence of the series, $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n+1}$. [5]

(b) Find the first four nonzero terms of the Taylor series generated by $f(x) = \dfrac{1}{1+x}$, at $x = 3$. [5]

END OF EXAMINATION