University of ESwatini

Supplementary/Resit Examination, January 2019

B.A.S.S. , B.Sc, B.Eng, B.Ed

Title of Paper : Calculus I

Course Number : M211/MAT211

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Begin each major question (A1, B2, etc) on a new page.
3. Each question in Section B is worth 20%.
4. Show all your working.
5. Non programmable calculators may be used (unless otherwise stated).
6. Special requirements: None.
7. Indicate your program next to your student ID number.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A: Answer All Questions

A1.

(a) i. State Rolle's Theorem. [3]

ii. Consider the function and interval, $f(x) = \left|\frac{1}{x}\right|, \quad [-1,\ 1]$.
Explain why Rolle's Theorem does not apply to the function even though there exists a and b such that $f(a) = f(b)$. [4]

iii. Find and classify all critical points of the function, $y = -2x^3 + 6x^2 - 3$. [6]

iv. Use L'Hôpital's Rule to show that $\lim_{x \to 0^+} \sqrt{x} \ln x = 0$. [3]

v. Express the function $f(x) = \dfrac{2x^2 - 4x}{x+1}$ as a sum of its quotient and remainder, hence show that the graph of $f(x)$ has a slant asymptote at the line $2x - 6$. [4]

(b) Set up the integral for finding the area of the region bounded by the graphs of $y_1 = 3(x^3 - x)$ and $y_2 = 0$. DO NOT EVALUATE. [4]

(c) i. Simplify the ratio of factorials, $\dfrac{(2n-1)!}{(2n+1)!}$. [3]

ii. List the first five terms of the sequence $a_n = \dfrac{3n}{n+4}$. [3]

iii. In your own words, define a monotonic sequence. [3]

iv. Define a $p-$series and state the requirements for its convergence. [4]

v. State the Ratio Test for series absolute convergence. [3]

Section B: Answer Three(3) Questions Only

B2.

Consider the function $f(x) = \frac{x^2+4}{2x}$.

(a) Identify the domain of $f(x)$. [1]

(b) Find and **classify** all critical points of f. [4]

(c) Find intervals where f is increasing and where it is decreasing. [4]

(d) Find possible points of inflection, if any occur and determine concavity of the graph. [3]

(e) Identify any asymptotes that may exist. [3]

(f) Sketch the graph of f labelling all major points found above including intercepts if any occur. [5]

B3.

(a) Sketch and find the area of the region bounded by the graphs of $x = y^3$ and $x = y^2$ [10]

(b) Sketch the region bounded by the curves $y = x^2 - 4x + 5$, $x = 1$, $x = 4$, and the $x-$axis. Use the **Disk Method** to determine the volume of the solid obtained by rotating the region about the x-axis. [10]

B4.

(a) Find the arc length of the graph of $x = \frac{1}{3}(y^2+2)^{\frac{3}{2}}$ over the interval $0 \leq y \leq 4$. [10]

(b) Find the area of the surface generated by revolving the curve, $y = \frac{x^3}{9}$, $0 \leq x \leq 2$, about the $x-$axis. [10]

B5.

(a) Consider the sequence and its first term, $a_{n+1} = \dfrac{n\, a_n}{n+1}$, $a_1 = -2$.
Find the values of a_2, a_3 and a_4. [3]

(b) Determine whether the sequence with the given nth term is monotonic and whether it is bounded.

i. $a_n = 4 - \dfrac{1}{n}$. [3]

ii. $a_n = n\, e^{-\frac{n}{2}}$. [4]

(c) Write an expression for the nth term of the sequence $1, -4, 9, -16, 25, \ldots$. [3]

(d) Determine the convergence or divergence of the sequence $a_n = \left(1 - \dfrac{1}{n}\right)^n$.
If the sequence converges, find its limit. [7]

B6.

(a) Express the repeating decimal 5.23232323... as a ratio of two integers. [5]

(b) Using a Test of your choice, show that the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$, diverges. [5]

(c) Find the radius of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(4x)^n}{n^2}$. [5]

(d) Find the 4–th Maclaurin polynomial for the function $f(x) = e^{-\frac{x}{2}}$ [5]

END OF EXAMINATION