University of Eswatini

Final Examination, May 2019

B.Sc II, B.A.S.S II, B.Ed II, B.Eng II

Title of Paper : Calculus II

Course Code : MAT212/M212

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) (i) Sketch the curve described by the parametric equations
$x = t^2 - 4,\ y = \frac{t}{2}, -2 \leq t \leq 3.$ [4]

(ii) For the curve given by $x = \sqrt{t}$ and $y = \frac{1}{4}(t^2 - 4),\ t \geq 0$ find the slope and concavity at the point (2, 3). [4]

(iii) Skecth the polar curve
$r = \sec\theta$ also find a Cartesian equation for this curve. [4]

(b) (i) Find $\lim_{(x,y)\to(1,2)} \frac{5x^2y}{x^2+y^2}$. [4]

(ii) Find the total differential of the function $z = 2x\sin y - 3x^2y^2$. [4]

(iii) Find the directional derivative of the function
$f(x,y) = 4 - x^2 - \frac{1}{4}y^2$ at $(1,2)$ in the direction of the vector $\mathbf{v} = (\cos\frac{\pi}{3})\mathbf{i} + (\sin\frac{\pi}{3})\mathbf{j}$. [5]

(c) (i) Calculate the double integral
$\iint_R \sin(x-y)dA, R = \{(x,y) : 0 \leq x \leq \pi/2, 0 \leq y \leq \pi/2\}$. [5]

(ii) Evaluate the iterated integral $\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} \sin(x^2+y^2)dydx$ by converting to polar coordinates. [5]

(iii) Evaluate the iterated integral $\int_0^{\pi/2}\int_0^y\int_0^x \cos(x+y+z)dzdxdy$. [5]

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

(a) Find the exact length of the polar curve
$r = \theta^2, \quad 0 \leq \theta \leq 2\pi.$ [10]

(b) Find the area of the region that lies inside the circle $r = 3\sin\theta$ and outside the cardioid $r = 1 + \sin\theta$. [10]

Question 3

(a) Find the horizontal and vertical tangent lines of $r = \sin\theta$, where $0 \leq \theta < \pi$. [10]

(b) Sketch the the graph of $r = 2\cos 3\theta$. [10]

Question 4

(a) Find the limit, if it exists, or show that the limit does not exist for
$$\lim_{(x,y)\to(0,0)} \frac{y^2 \sin^2 x}{x^4 + y^4}.$$ [10]

(b) Find the local maximum and minimum values and saddle point(s), if any, of the function
$$f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$$ [10]

Question 5

(a) Use the Chain Rule to find the indicated partial derivatives.
$z = x^4 + x^2y,\ x = s + 2t - u, y = stu^2$
$\frac{\partial z}{\partial s}, \frac{\partial z}{\partial t}, \frac{\partial z}{\partial u}$ when $s = 4, t = 2, u = 1$ [10]

(b) Find the equation of the tangent plane to the hyperboloid $z^2 - 2x^2 - 2y^2 = 12$ at the point (1, -1, 4). [10]

Question 6

(a) Find the mass and center of mass of a triangular lamina with vertices $(0,0), (1,0)$ and $(0,2)$ if the density function is $\rho(x,y) = 1 + 3x + y$. [10]

(b) Let R be the annular region lying between the two circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 5$. Evaluate the integral
$\int\int_R (x^2 + y)dA$. [10]

End of Examination Paper