University of Eswatini

Re-sit/Supplementary Examination, July 2019

B.Sc II, B.A.S.S II, B.Ed II, B.Eng II

Title of Paper : Calculus II

Course Code : MAT212/M212

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) (i) Sketch the curve represented by the equations

$$x = \frac{1}{\sqrt{t+1}} \text{ and } y = \frac{t}{t+1}, t > -1.$$ [4]

(ii) Find $\frac{dy}{dx}$ for the curve given by $x = \sin t$ and $y = \cos t$. [4]

(iii) Sketch the polar curve $\theta = \frac{\pi}{3}$ also find a Cartesian equation for this curve. [4]

(b) (i) Find the domain of $f(x, y) = \frac{\sqrt{x^2 + y^2 - 9}}{x}$. [5]

(ii) Discuss the continuity of the function $f(x, y) = \frac{2}{y - x^2}$. [5]

(iii) Find the directional derivative of $f(x, y) = x^2 \sin 2y$ at $(1, \frac{\pi}{2})$ in the direction of $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$. [6]

(c) (i) Sketch the region whose area is represented by the integral $\int_0^2 \int_{y^2}^4 dxdy$.
Then find another iterated integral using the order $dydx$ to represent the same area and show that both integrals yield the same value. [6]

(ii) Evaluate the iterated integral $\int_0^2 \int_0^{z^2} \int_0^{y-z} (2x - y)dxdydz$. [6]

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

(a) Find the horizontal and vertical tangent lines of
$r = \sin\theta$ where $0 \leq \theta < \pi$. [10]

(b) Find the area of the region that is bounded by the given curve and lies in the specified sector
$$r^2 = 9\sin 2\theta, \quad r \geq 0, \ 0 \leq \theta \leq \pi/2.$$ [10]

Question 3

(a) Find an equation of the tangent to the curve at the given point by two methods: (i) without eliminating the parameter and (ii) by first eliminating the parameter. If the curve is given by
$$x = 1 + \ln t, \quad y = t^2 + 2, \quad (1, 3).$$ [10]

(b) Find the length of the arc from 0 to 2π for the cardioid $r = 2 - 2\cos\theta$. [10]

Question 4

(a) Find the limit, if it exists, or show that the limit does not exist for
$$\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}.$$ [10]

(b) Find the local maximum and minimum values and saddle point(s), if any, of the function
$$f(x, y) = (x - y)(1 - xy).$$ [10]

Question 5

(a) Find the differential of the function

$$z = e^{-2x}\cos 2\pi t.$$ [10]

(b) Find the equation of the tangent plane to the paraboloid

$$z = 1 - \frac{1}{10}(x^2 + 4y^2)$$ at the point $(1, 1, \frac{1}{2})$. [10]

Question 6

(a) Evaluate the double integral

$$\int\int_R x dA$$

where D is the region given by $D = (x, y)|\ 0 \leq x \leq \pi,\ 0 \leq y \leq \sin x.$ [10]

(b) Use an iterated integral to find the area of the region bounded by graphs of the $f(x) = \sin x$ and $g(x) = \cos x$ between $\frac{\pi}{4} \leq x \leq \frac{5\pi}{4}$. [10]

End of Examination Paper