University of eSwatini

Final Examination, May 2019

B.Sc , BASS, B.Ed, B.Eng

Title of Paper : Ordinary Differential Equations

Course Code : MAT216/M213

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Special requirements: None

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A: Answer ALL Questions

A1. **a.** By eliminating A and B, determine the ODE satisfied by the function, $y = Ae^{2x} + Be^{-x}$. [3]

b. Find an integrating factor for

$$2xy^3dx + (3x^2y^2 + x^2y^3 + 1)dy = 0.$$

[3]

c. Solve the ODE,

$$(x^2 + 3xy + y^2)dx - x^2dy = 0.$$

[6]

d. Find the second, linearly independent solution of $x^2y'' + 2xy' - 6y = 0$, given that $y(x) = x^2$ is a solution of the ODE. [8]

e. Solve the initial value problem,

$$y'' - 7y' + 10y = 0, \quad y(0) = k_0, \quad y'(0) = k_1.$$

[5]

f. Find the general solution of,

$$y^{iv} + 4y''' + 6y'' + 4y' = 0.$$

[6]

g. Find the inverse Laplace transform of,

$$F(s) = \frac{2 + 3s}{(s^2 + 1)(s + 2)(s + 1)}.$$

[5]

h. Reduce the following ODE into a system of first order ODEs, leaving your answer in matrix form.

$$\ddot{y} + 5\dot{y} + 6y = 0.$$

[4]

Section B: Answer ANY 3 Questions

B2. (a) Show that if $\dfrac{N_x - M_y}{M} = Q$, where Q is a function of y only, then the differential equation,

$$M(x, y)dx + N(x, y)dy = 0$$

has an integrating factor of the form

$$\mu(y) = \exp\left(\int Q(y)dy\right).$$

[10]

(b) Find the value of b for which $(xy^2 + bx^2y)dx + (x + y)x^2dy = 0$ is exact, and then solve it using that value of b. [10]

B3. (a) Verify that $y_1(x) = e^x$ and $y_2(x) = x$ satisfy the corresponding homogeneous equation of the nonhomogeneous equation,

$$(1 - x)y'' + xy' - y = 2(x - 1)^2e^{-x}.$$

[5]

(b) Hence use the method of variation of parameters to find a particular solution of the nonhomogeneous equation. [15]

B4. Solve the given differential equation by means of a power series about $x_0 = 0$. Find the recurrence relation and the first four terms in each of two linearly independent solutions.

$$(1 + 2x^2)y'' + 6xy' + 2y = 0.$$

[20]

B5. (a) Find the Laplace transform of

$$f(t) = \cosh^2 t.$$

[5]

(b) Solve the following IVP using Laplace transform.

$$y'' - 6y' + 5y = 3e^{2t}, \quad y(0) = 2, \quad y'(0) = 3.$$

[15]

B6. (a) Reduce the following ODE into a system of first order ODEs, leaving your answer in matrix form.

$$6\ddot{y} + \dot{y} - 5y = 0.$$

[5]

(b) Solve the initial value problem given

$$\frac{d\mathrm{X}}{dt} = \begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \end{pmatrix} = A\mathrm{X}, \quad \text{where,} \quad A = \begin{pmatrix} 21 & -12 \\ 24 & -15 \end{pmatrix}, \quad \text{given,} \quad \mathrm{X}(0) = \begin{pmatrix} 5 \\ 3 \end{pmatrix}.$$

[15]

END OF EXAMINATION

Table 1: Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1	$\frac{1}{s}, \quad s > 0.$
e^{at}	$\frac{1}{s-a}, \quad s > a.$
$t^n, \quad n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0.$
$t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0.$
$\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0.$
$\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0.$
$\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \|a\|.$
$\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \|a\|.$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a.$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a.$
$t^n e^{at}, \quad n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a.$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \cdots - f^{(n-1)}(0)$
$(-t)^n f(t)$	$F^{(n)}(s)$