University of eSwatini

Supplementary/Resit Examination, July 2019

B.Sc , BASS, B.Ed, B.Eng

Title of Paper : Ordinary Differential Equations

Course Code : MAT216/M213

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.

   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.

   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.

2. Each question in Section B is worth 20%.

3. Show all your working.

4. Special requirements: None

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A: Answer ALL Questions

A1. **a.** Identify the non-linear term in each of the following non-linear ODEs.

i. $\dfrac{dy(x)}{dx} - \cos(y(x)) = \sin(x)$ [1]

ii. $\dfrac{d^2x(t)}{dt^2} - 5\dfrac{dx(t)}{dt}(x(t)) = e^t$ [1]

iii. $(1+x^2)\dfrac{d^2y(x)}{dx^2} - \left|\dfrac{dy(x)}{dx}\right| = 0$ [1]

b. Find the particular solution of the following IVP given that its general solution is $y^2 = 1 + Ae^{x^2}$.

$$yy' - xy^2 + x = 0, \quad y(-1) = -1.$$

[4]

c. Solve the initial value problem, $y' + \left(\dfrac{1+x}{x}\right) y = 0, \quad y(1) = 1.$ [5]

d. Verify that $y_1 = e^x \cos x$ and $y_2 = e^x \sin x$ are solutions of

$$y'' - 2y' + 2y = 0, \quad \text{on} \quad (-\infty, \infty).$$

[5]

e. Find the particular solution for the IVP,

$$y'' - 3y' + 2y = 0, \quad y(0) = -2, \quad y'(0) = 2.$$

[7]

f. Find the general solution of the ODE,

$$y^{iv} + y''' - 7y'' - y' + 6y = 0.$$

[6]

g. Find the inverse Laplace transform of

$$F(s) = \frac{2+3s}{s^2 - 3s + 2.}$$

[5]

h. Reduce the following ODE into a system of first order ODEs, leaving your answer in matrix form.

$$\ddot{y} + 2\dot{y} - 3y = 0.$$

[5]

Section B: Answer ANY 3 Questions

B2. (a) Confirm that

$$(4xy^2 + 3y)dx + (3x^2y + 2x)dy = 0$$

has an integrating factor of the form $\mu = x^m y^n$. Determine m and n, hence solve the ODE. [13]

(b) Solve the Bernoulli equation

$$y' + y = y^2 e^x.$$

[7]

B3. (a) Use the method of undetermined coefficients to find the general solution of

$$y'' + 2y' + y = 8x^2 \cos x - 4x \sin x.$$

[15]

(b) Let $\phi_1(x)$ and $\phi_2(x)$ be any two differentiable functions. Prove that if the *Wronskian* vanishes, then one function is a constant multiple of the other. [5]

B4. Suppose

$$y = \sum_{n=0}^{\infty} a_n (x-1)^n$$

on an open interval I that contains $x_0 = 1$. Express the function

$$(1+x)y'' + 2(x-1)^2 y' + 3y$$

as a power series in $(x-1)$ on I. Find the recurrence relation. [20]

B5. (a) Find the Laplace transform of

$$f(t) = t\sinh(2t).$$

[5]

(b) Solve the following IVP using Laplace transform.

$$y'' - 3y' + 2y = e^{3t}, \quad y(0) = 1, \quad y'(0) = -4.$$

[15]

B6. (a) Reduce the following ODE into a system of first order ODEs, leaving your answer in matrix form.

$$3\ddot{y} + 5\dot{y} + 2y = 0.$$

[5]

(b) Find the general solution of

$$\frac{d\mathbf{X}}{dt} = \begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \end{pmatrix} = A\mathbf{X}, \quad \text{where,} \quad A = \begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix}.$$

[15]

END OF EXAMINATION

Table 1: Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1	$\frac{1}{s}, \quad s > 0.$
e^{at}	$\frac{1}{s-a}, \quad s > a.$
$t^n, \quad n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0.$
$t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0.$
$\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0.$
$\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0.$
$\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \|a\|.$
$\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \|a\|.$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a.$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a.$
$t^n e^{at}, \quad n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a.$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
$(-t)^n f(t)$	$F^{(n)}(s)$