University of Eswatini

Resit/Supplementary Examination, 2018/2019

B.A.S.S. II, B.Ed (Sec.) II, B.Sc. II, B.Eng. II

Title of Paper : Linear Algebra

Course Number : MAT221/M220

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

Special Requirements: NONE

This examination paper should not be opened until permission has been given by the invigilator.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

a) i) Determine whether the vectors $\bar{v}_1 = (4, 1, -2)$, $\bar{v}_2 = (-3, 0, 1)$, and $\bar{v}_3 = (1, 2, 1)$ are linearly independent or linearly dependent. [4]

ii) Let P_{nn} be the vector space of $n \times n$ matrices. Determine whether the transformation $T(A) = A^T - 3A$ is linear transformation or not. [4]

b) i) Find $|A|$ and $|A^T|$, given that [4]

$$A = \begin{bmatrix} \pi & 6 & 1 & -4 \\ 0 & -3 & 0 & 7 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

ii) If C is a 4 by 4 matrix, and $|C| = 3$, determine $|4C|$. [2]

c) i) Determine the characteristic polynomial of [2]

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 3 & 0 & 0 \\ -4 & 0 & -3 \end{bmatrix}$$

ii) Hence find the corresponding eigenvalues and Eigenvectors. [6]

d) i) Express [4]

$$C = \begin{bmatrix} -1 & 1 \\ 0 & 3 \end{bmatrix}$$

as a product of elementary matrices.

ii) Find $2A^8 - 3I$, where [4]

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

e) Solve the following system, where a, and b are constants using Gauss-Jordan elimination. [4]

$$2x_1 + x_2 = 18$$
$$3x_1 + 6x_2 = 9$$

f) Verify the Cayley-Hamilton theorem for the matrix [6]

$$A = \begin{bmatrix} 1 & 4 \\ 0 & -3 \end{bmatrix}$$

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

a) Solve the system of equations [12]

$$\begin{aligned} x_1 - 2x_2 + 5x_3 &= 2 \\ 4x_1 - 5x_2 + 8x_3 &= 3 \\ -3x_1 + 3x_2 - 3x_3 &= -1. \end{aligned}$$

b) If A is an invertible matrix, prove that A^T is also invertible and $(A^{-1})^T = (A^T)^{-1}$. [8]

QUESTION B3 [*20 Marks*]

a) Suppose that $\det(A^{-1}) = -10$. Find $\det(A)$. [5]

b) Consider the linear system of equations

$$\begin{aligned} kx_2 + x_3 &= 1 \\ x_1 + x_2 + x_3 &= 1 \\ x_1 + 2x_2 + (1+k)x_3 &= 2. \end{aligned}$$

i) Determine the value(s) of k for which the system has infinitely many solutions. [5]

ii) Find the solution(s) of the system for the value(s) of k. [10]

QUESTION B4 [*20 Marks*]

a) State Cayley-Hamilton theorem. [5]

b) Find bases for the eigenspaces of the matrix [10]

$$A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$$

c) Consider the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

i) Find the eigenvalues of the matrix A. [3]

ii) Is the matrix A invertible or not? [2]

QUESTION B5 [*20 Marks*]

a) Let V_{nn} be the vector space of $n \times n$ matrices. Determine whether the transformation $T(A) = \det(A)$ is linear. [4]

b) Prove that If $T : V \to W$ is a linear transformation, then:

i) $T(\mathbf{0}) = \mathbf{0}$ [4]

ii) $T(\mathbf{u}) = -T(\mathbf{u})$ [4]

iii) $T(\mathbf{u} - \mathbf{v}) = T(\mathbf{u}) - T(\mathbf{v})$. [8]

QUESTION B6 [*20 Marks*]

a) Determine whether the polynomials [10]

$$\mathbf{p}_1 = 2 - 2x, \quad \mathbf{p}_2 = 10 + 6x - 4x^2, \quad \mathbf{p}_3 = 2 + 6x - 2x^2$$

are linearly dependent or linearly independent in $\mathbb{P}_2$.

b) Do the vectors $\mathbf{v}_1 = (1, 2, 1)$, $\mathbf{v}_2 = (2, 9, 0)$, $\mathbf{v}_3 = (3, 3, 4)$, form a basis for $\mathbb{R}^3$? [10]

END OF EXAMINATION PAPER