UNIVERSITY OF ESWATINI

RESIT EXAMINATION, 2018/2019

BASS, B.Ed (Sec.), B.Sc.

Title of Paper : Foundations of Mathematics

Course Number : MAT231/M231

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SEVEN (7) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, A2, B2, ..., B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*20 Marks*]

(a) Determine whether or not the given sentence is a proposition. If it is a proposition, give its truth value. (6)

i. $\exists x \in \mathbb{R}, x^2 - 2x + 1 > 0$.

ii. Are you at home now?

iii. Mbabane is a city in Eswatini.

iv. x is a real number.

(b) Give clear definitions of each of the following

i. An *equivalence relation* on a set A? (4)

ii. An *injective function* from a set A into a set B. (2)

iii. A *surjective function* from a set A into a set B. (2)

(c) Write down (i.) the inverse, (ii.) the converse, and (iii.) the contrapositive of the following statement.

$$\neg(p \vee q) \to r.$$

(6)

QUESTION A2 [*20 Marks*]

(a) Let $\mathbb{R}^+$ be the set of positive real numbers. True or False? (Explain your answer). (3)

$$\forall x \in \mathbb{R}^+, x > \frac{1}{x}.$$

(b) Write down the negation of the proposition

$$\forall x \in \mathbb{R}, \text{if } x(x+1) > 0, \text{ then } x > 0 \text{ or } x > -1.$$

(5)

(c) Use a truth table to determine whether or not the following argument is valid. (6)

$$\begin{array}{rl} & p \to q \\ & q \to r \\ \hline \therefore & p \to r \end{array}$$

(d) Without using truth tables, show that $\neg p \vee (p \wedge q) \equiv p \to q$. (6)

END OF SECTION A – TURN OVER

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B3 [20 *Marks*]

(a) Prove: *For all integers n, if n is odd, then n^2 is odd.* (4)

(b) Prove: *For an integer n, if $n^3 + 5$ is odd, then n is even.* (5)

(c) Prove: *If n is an odd integer, then there exists an integer m such that $n^2 = 8m + 1$.* (7)

(d) Let $a, b, c \in \mathbb{Z}, a \neq 0, b \neq 0$. Prove: *If $a \mid b$ and $a \mid c$, then $a \mid (b + c)$.* (4)

QUESTION B4 [20 *Marks*]

(a) i. Define a *partition* of a set A. (2)

ii. Let $A = \{1,2,3,4,5,6\}, A_1 = \{1\}, A_2 = \{2,3\}, A_3 = \{4,5\}$. Does $\{A_1, A_2, A_3\}$ form a partition of A? (2)

(b) Let A and B be sets in a universal set U. Prove

i. If $A \subseteq B$, then $A \cup B = B$. (5)

ii. $(A \setminus B) \cap B = \emptyset$. (5)

ii. $(A \cap B)^c = A^c \cup B^c$. (6)

QUESTION B5 [20 *Marks*]

(a) Use mathematical induction to prove that $2^{3n} - 1$ is divisible by 7 for all integers $n \geq 1$. (7)

(b) Use strong induction to prove: *Any integer $n > 1$ is either a prime number or can be written as a product of prime numbers.* (7)

(c) Find a solution to the sequence recursively defined by

$$a_1 = 1,\ a_2 = 2, \quad a_n = 2a_{n-1} + 3a_{n-2},\ n \geq 3.$$

(6)

QUESTION B6 [20 *Marks*]

(a) Let $A = \{a, b, c, d\}$ and $B = \{1, 2, 3, 4, 5, 6, 7\}$. Which of the following relations from A into B are functions? Explain your answer. (4)

i. $\{(a, 4), (d, 3), (b, 5), (c, 2), (a, 6)\}$

iii. $\{(a, 1), (b, 1), (c, 1), (d, 1)\}$.

(b) Find the domain and range of each function below. (4)

i. $f(x) = \ln |x|$. ii. $g(x) = \sqrt{3 - x}$.

(c) Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^3 - 1$ and $g(x) = \sqrt[3]{x + 1}$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$. (4)

(d) Let $f : \mathbb{R} \to \mathbb{Z}$ be defined by $f(x) = \lfloor x \rfloor$ (the greatest integer less than or equal to x). Show that f is not injective. (2)

(e) Let $f : A \to B$ and $g : B \to C$ be injective functions. Show that $g \circ f : A \to C$ is also injective. (6)

QUESTION B7 [20 *Marks*]

(a) Let $A = \{2, 4\}$, $B = \{6, 8, 10\}$. Define the binary relations R and S from A to B as follows:

$$(x, y) \in R \text{ if and only if } x \mid y$$
$$(x, y) \in S \text{ if and only if } y - 4 = x.$$

List the elements of R and S. (4)

(b) Let $A = \{1, 2, 3, 4\}$ and define a relation R on A by xRy if and only if $x \leq y$. Find the domain and range of R. (2)

(c) Define a relation R on $\mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$ as follows: $(m, n)R(m', n')$ if and only if $mn' = nm'$. Show that R is an equivalence relation on $\mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$. (8)

(d) Let $\mathscr{A}$ be a collection of sets. Let R be the relation on $\mathscr{A}$ defined as follows: For $A, B \in \mathscr{A}$, $(A, B) \in R$ if and only if $A \subseteq B$. Show that R is a partial order on $\mathscr{A}$. (6)

END OF EXAMINATION PAPER