University of Swaziland

Main Examination, 2018/2019

BASS, B.Ed (Sec.), B.Sc.

Title of Paper : Dynamics I

Course Number : MAT256/M255

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SEVEN (7) questions in TWO sections.
2. Section A consists of TWO questions and is **COMPULSORY**. Each question is worth 20%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, A2, B1 – B7) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*20 Marks*]

(a) Let $\mathbf{a} = -2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{b} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{k}}$. Find a unit vector perpendicular to both $\mathbf{c} = \mathbf{b} - \mathbf{a}$ and $\mathbf{d} = 2\mathbf{a} + \mathbf{b}$. (5)

(b) Let $\mathbf{r}(t) = \langle 3\cos 4t, 5t, \sin t\rangle$. Find $\hat{\mathbf{T}}(0)$. (4)

(c) Show that if $|\mathbf{r}(t)| = c$, where c is a constant, then $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are perpendicular. (5)

(d) A particle has acceleration given by $\mathbf{a}(t) = 2\hat{\mathbf{i}} + \hat{\mathbf{j}}$. Given that $\mathbf{v}(0) = \hat{\mathbf{k}}$ and $\mathbf{r}(0) = \frac{1}{2}\hat{\mathbf{j}}$, find the position vector of the particle at any time t. (6)

QUESTION A2 [*20 Marks*]

(a) A particle moves along the space curve defined by

$$\mathbf{r}(t) = 3t\hat{\mathbf{i}} + 4\sin t\hat{\mathbf{j}} + 4\cos t\hat{\mathbf{k}}.$$

Find

i. the velocity of the particle, (2)

ii. the speed of the particle, (2)

iii. the acceleration of the particle, (2)

iv. the curvature κ and the radius of curvature R, (2)

v. the unit principal normal $\hat{\mathbf{N}}$, (2)

vi. the tangential component of acceleration, (2)

vii. the normal component of acceleration. (2)

(b) At time t, the speed v of a particle moving in a straight line is given by

$$v = \frac{(5 - t^2)x^2}{10t^2}$$

where x is the distance covered after time t. If at time $t = 1$, the particle has covered a distance $x = 5/3$, find an expression for x in terms of t. (6)

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B3 [*20 Marks*]

(a) A force with magnitude 20 N acts in the direction of the positive z-axis on an object with mass 5 kg. The object starts at the origin with initial velocity $\mathbf{v}(0) = \hat{\mathbf{i}} - \hat{\mathbf{j}}$ m/s. Find its position vector and its speed at time t. (6)

(b) A body is projected vertically upward with speed 20 m/s. Find the greatest height reached and the time taken to reach this height. [Take $g = 10$ m/s^2]. (7)

(c) A body of mass m falls from rest from a height h above the ground. Show that it strikes the ground after a time $\sqrt{\dfrac{2h}{g}}$ with speed $\sqrt{2gh}$. (7)

QUESTION B4 [*20 Marks*]

(a) The acceleration (in m/s^2) of a particle is given by

$$a = \frac{5 + 4v^2}{8},$$

where v is its speed (in m/s) at a distance x metres from the origin. If the particle started from rest at the origin, determine how far it has travelled when it reaches a speed of 2 m/s. (7)

(b) At time $t = 0$, a particle of mass m is located at $z = 0$ and is travelling vertically downward with speed v_0. Suppose the particle experiences a resisting force of magnitude kv where v is the speed of the particle and k is a positive constant. Find

i. the speed of the particle at any time $t > 0$, (5)

ii. the distance travelled at time $t > 0$, (5)

iii. the acceleration of the particle at time $t > 0$. (3)

QUESTION B5 [*20 Marks*]

(a) A projectile is fired with an initial speed of 200 m/s and an angle of elevation 45°. Assuming $g = 10$ m/s^2, find

i. the velocity vector of the projectile at any time t, (3)

ii. the position vector of the projectile at any time t, (3)

iii. the range of the projectile, (3)

iv. The maximum height reached. (3)

(b) A particle is projected from a point A with speed 20 m/s at an angle θ to the horizontal. It passes through a point B which is at horizontal distance 30 metres from A and at a vertical height 8.75 metres above A. Take $g = 10$ m/s^2. Show that

$$\tan\theta = \frac{4}{3}.$$

(8)

QUESTION B6 [*20 Marks*]

(a) A train journey takes a total time T from rest to rest. It travels with uniform acceleration for a time T/n (where n is a positive integer), then with uniform speed V for a time $(n-2)T/n$, and finally, for time T/n with constant retardation.

i. Prove that the average speed of the train is given by

$$(n-1)\frac{V}{n}$$

(8)

ii. Suppose that the length of the journey is 64 km, the total time taken is 50 minutes, and the uniform speed is 96 km/h. Find the duration of travel under uniform speed. (4)

(b) A car with initial speed u accelerates uniformly over a distance $2s$, which it covers in time t_1. It is then stopped by being retarded uniformly over a distance s, which it covers in time t_2. Prove that

$$\frac{u}{2s} = \frac{2}{t_1} - \frac{1}{t_2}.$$

(8)

QUESTION B7 [*20 Marks*]

(a) Show that

$$x(t) = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t$$

can be written as

$$x(t) = A \cos\left(\omega_0 t + \phi\right)$$

where $A = \sqrt{c_1^2 + c_2^2}$ and $\phi = \arctan\left(-c_2/c_1\right)$. (6)

(b) Find the amplitude and phase constant for the block-spring system with position given by

$$x(t) = \sqrt{3} \cos \omega_0 t - \sin \omega_0 t.$$

(4)

(c) When a 4 kg mass attached to a spring, it is observed to oscillate with a period of 2 seconds. Find the period of oscillation if a 6 kg mass is attached to the spring. (5)

(d) A simple pendulum of length 0.6 m with a block of mass 5 kg has a maximum speed of 1.2 m/s. Find the maximum height reached by the block. (Assume $g = 10$ m/s^2.) (5)

END OF EXAMINATION PAPER