University of Eswatini

Re-Sit Examination, 2018/2019

BASS, B.Ed (Sec.), B.Sc.

Title of Paper : Dynamics I

Course Number : MAT256/M255

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SEVEN (7) questions in TWO sections.
2. Section A consists of TWO questions and is **COMPULSORY**. Each question is worth 20%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, A2, B3 – B7) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*20 Marks*]

(a) Let $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{b} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$. Find a unit vector perpendicular to both $\mathbf{c} = \mathbf{a} - \mathbf{b}$ and $\mathbf{d} = \mathbf{a} + \mathbf{b}$ (5)

(b) Let $\mathbf{r}(t) = \langle 3\cos t, 4t, 3\sin t\rangle$ define a space curve C. Find the unit normal to the curve at the point $(3, 0, 0)$. (5)

(c) Show that if $|\mathbf{r}(t)| = c$, where c is a constant, then $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are perpendicular. (5)

(d) A particle has acceleration given by $\mathbf{a}(t) = 2t\hat{\mathbf{i}} + \sin t\hat{\mathbf{j}} + \cos 2t\hat{\mathbf{k}}$. Given that $\mathbf{v}(0) = \hat{\mathbf{i}}$ and $\mathbf{r}(0) = \hat{\mathbf{j}}$, find the position vector of the particle at any time t. (5)

QUESTION A2 [*20 Marks*]

(a) A particle moves along the space curve defined by

$$\mathbf{r}(t) = 3\cos 2t\hat{\mathbf{i}} + 3\sin 2t\hat{\mathbf{j}} + (8t - 4)\hat{\mathbf{k}}.$$

Find

i. the velocity of the particle, (2)

ii. the speed of the particle, (2)

iii. the acceleration of the particle, (2)

iv. the curvature κ and the radius of curvature R, (3)

v. the unit principal normal $\hat{\mathbf{N}}$, (2)

vi. the tangential component of acceleration, (2)

vii. the normal component of acceleration. (2)

(b) At time t, the speed v of a particle moving in a straight line is given by

$$v = \frac{(9 - t^4)x^3}{3t^4}, \quad t > 0$$

where x is the distance covered after time t. If at time $t = 1$, the particle has covered a distance $x = 5/3$, find an expression for x in terms of t. (6)

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B3 [*20 Marks*]

(a) Particle A, initially at rest, is projected from the origin with acceleration $\frac{\sqrt{3}}{2}\hat{\mathbf{i}} + \frac{1}{2}\hat{\mathbf{j}}$. At the same instant, particle B at rest at the point $(\sqrt{3}, 0)$, is projected with acceleration $12\hat{\mathbf{j}}$. Show that the particles collide and find the time of collision. (8)

(b) A body is projected vertically upward with speed v_0. Find the greatest height reached and the time taken to reach this height. [Leave your answer in terms of g]. (6)

(c) A body of mass 2 kg falls from rest from a height 5 metres above the ground. Assuming $g = 10$ m/s^2, find

i. the time it hits the ground, (4)

ii. its speed when it hits the ground. (2)

QUESTION B4 [*20 Marks*]

(a) The acceleration (in m/s^2) of a particle is given by

$$a = \frac{10}{4 + 5\sqrt{v}},$$

where v is its speed (in m/s) at a distance x metres from the origin. If the particle started from rest at the origin, determine how far it has travelled when it reaches a speed of 16 m/s. (8)

(b) At time $t = 0$ a particle of mass 2 kg falling under gravity is positioned at the origin and is travelling vertically downward with speed 10 m/s. Suppose that the resisting force at speed v has magnitude $0.5v$ Newtons. Assuming $g = 10$ m/s^2, find the speed $v(t)$ and the distance travelled $z(t)$ at any time t. (12)

QUESTION B5 [*20 Marks*]

(a) A projectile is fired with an initial speed of v_0 at angle of elevation α. Find (in terms of α, v_0 and g)

i. the velocity vector of the projectile at any time t, (3)

ii. the position vector of the projectile at any time t, (3)

iii. the range of the projectile, (3)

iv. the maximum height reached, (3)

v. the value of α that will maximise the range of the projectile. (3)

(b) Suppose an object of mass m has position vector given by

$$\mathbf{r}(t) = a\cos\omega t + b\sin\omega t,$$

where a, b, and ω are constants. Find the force acting on the object and show that the force is directed towards the origin. (5)

QUESTION B6 [*20 Marks*]

(a) A train takes a time T to complete a journey from rest to rest. It accelerates uniformly from rest for a time pT and retards uniformly to rest at the end of the journey for a time qT. During the intermediate time, it travels uniformly with speed v. Prove that the average speed for the journey is

$$\frac{1}{2}v(2-p-q).$$

(8)

(b) Two points A and B are at a distance d apart. A particle starts from A and moves in the direction $\overrightarrow{AB}$ with initial velocity u and uniform acceleration a. At the same instant, a second particle starts from B and moves in the direction $\overrightarrow{BA}$ with initial velocity $2u$ and retardation a.

i. Show that the particles collide at time $\dfrac{d}{3u}$ after the start of motion. (8)

ii. Show that if the particles collide before the second particle returns to B, then $ad < 12u$. (4)

QUESTION B7 [*20 Marks*]

(a) Suppose one end of a spring with spring constant k and equilibrium length l is attached to a wall and an object of mass m is attached at the other end. Assume that the object moves in a straight line along a horizontal frictionless surface. Let the origin, O, be the position of the object at equilibrium position. Let the positive x direction be the direction when the spring is stretched, so that $x < 0$ means the spring is compressed. Suppose at time $t = 0$, $x(0) = x_0 > 0$ and the speed of the object is $v(0) = v_0$ away from O.

Show that the position x of the object at any time t is given by

$$x(t) = x_0 \cos \omega_0 t + \frac{v_0}{\omega_0} \sin \omega_0 t,$$

where $\omega_0 = \sqrt{k/m}$. (10)

(b) An object of mass 2 kg is attached to a spring with spring constant $k = 0.125$ kg per square second and is free to slide along a horizontal frictionless surface. At $t = 0$ the spring-object system is stretched an amount $x_0 = 0.1$ metres from the equilibrium position and is released from rest. Find the period of oscillation of the object and find its velocity when it first returns to the equilibrium position. (5)

(c) A simple pendulum of length 0.8 m with a block of mass 5 kg has a maximum speed of 1.2 m/s. Find the maximum height reached by the block. (Assume $g = 10$ m/s^2.) (5)

END OF EXAMINATION PAPER