UNIVERSITY OF ESWATINI

RESIT/SUPPLEMENTARY EXAMINATION, 2018/2019

BASS III, B.Ed (Sec.) III, B.Sc. III, B.Eng. III

Title of Paper : Complex Analysis

Course Number : MAT313/M313

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

a) Consider the complex number $\phi = -4 + 4i$. Determine the following:

i) Complex conjugate of ϕ. [2]

ii) Modulus of ϕ. [2]

iii) $Im(\phi - \bar{\phi})$ [2]

iv) $\ln(\phi)$. [4]

v) Principal value of the argument of ϕ. [2]

b) Determine the order of each pole of $f(z) = \frac{z}{z^2+1}$ and the corresponding residues. [5]

c) Determine whether $w = z^2(2 - 3i)$ is regular or not. [5]

d) Using the precise definition of a limit, show that

$$\lim_{z\to 2}\left(\frac{iz}{2}\right) = i.$$

[4]

e) Using the known Maclaurin series for $f(z) = \cos(z)$, find the Maclaurin series of

$$f(z) = z^3\cos(z^2).$$

[4]

f) Let C be a positively oriented circle such that $|z - 2i| = 2$. Evaluate

$$\int_C \frac{z-6}{(z+3i)(z-3i)}dz$$

[5]

g) Evaluate $\displaystyle\int_C \frac{dz}{z^2+4}$ where C is a semi-circle in the upper half plane of radius six. [5]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

a) Determine if the function $g(z) = 3x^2 + 2x - 3y^2 - 1 + i(6xy + 2y)$ is analytic everywhere or not? If $g(z)$ is analytic, find $g'(z)$. [10]

b) Prove that if a function $\phi(z) = \alpha(x, y) + i\beta(x, y)$ is analytic in a domain D, then $\alpha(x, y)$ and $\beta(x, y)$ are harmonic in D. [10]

QUESTION B3 [*20 Marks*]

a) Find the value of $\cosh\left(i - \frac{\pi}{2}\right)$ and express your answer in the form $a + ib$. [4]

b) Consider the equation $z^{2i} = 4$. Solve for z, and find $Im(z)$. [6]

c) Show that $\sin^{-1}(z) = -i\ln(iz + \sqrt{1 - z^2})$ [10]

QUESTION B4 [*20 Marks*]

a) Evaluate $\int_C \frac{4z^5}{(z-3)^3} dz$ if C is the circle $|z + 3| = 9$ [10]

b) State and prove Liouville's theorem. [10]

QUESTION B5 [*20 Marks*]

a) Find the Laurent series of $f(z) = \frac{2}{z(z-1)}$ in the domain $0 < |z| < 1$. [8]

b) Show that the Maclaurin series of $\sin(z) = \sum_{k=0}^{\infty} (-1)^k \frac{z^{2k+1}}{(2k+1)!}$ [12]

QUESTION B6 [*20 Marks*]

a) Evaluate $\int_C \frac{\cos(\pi z)}{z - 2} dz$ if C is a positively oriented circle such that $|z| = 3$. [4]

b) Using Cauchy's Residue Theorem, evaluate $\int_0^{\infty} \frac{2dx}{x^2 + 4}$. [8]

c) Let C be a positively oriented circle such that $|z| = 4$. Using Cauchy's residue theorem, evaluate [8]

$$\int_C \frac{z-2}{(z+1)(z^2+4)} dz$$

END OF EXAMINATION PAPER