University of Eswatini

Re-sit/Supplementary Examination, January 2019

B.Sc III, B.A.S.S III, B.Ed III

Title of Paper : Real Analysis

Course Code : MAT331/M331

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.

   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.

   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.

2. Each question in Section B is worth 20%.

3. Show all your working.

4. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) Define the following terms

(i) Limit point of a subset S of $\Re$. [2]

(ii) Bounded function. [2]

(b) (i) If x and y are any two real numbers, then prove that $|x+y| \leq |x| + |y|$ [4]

(ii) Prove that every subset of a bounded set is bounded. [4]

(iii) Using the ϵ, N definition show that the sequence $\left\{\frac{1}{n}\right\}$ converges to 0. [4]

(iv) Find the limit superior and limit inferior of sequence $< 1, 3, 5, 1, 3, 5, \cdots >$. [4]

(v) Show that the function $f(x) = 3x + 2$ is continuous in the interval $(0, 4)$. [4]

(vi) State Cauchy Criterion for convergent Series. [4]

(vii) If a function f is uniformly continuous on an interval I, then it is continuous on I. [4]

(viii) Prove that if a function is differentiable at a point then it is continuous at that point. [4]

(ix) State the Riemann's integrability criterion. [4]

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

(a) Define a Cauchy Sequence. [4]

(b) Show that if x is a limit point of A and $A \subset B$, then x is also a limit point of B. [8].

(c) By finding the left-hand and right-hand derivatives of

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

determine $f'(0)$. [8]

Question 3

(a) Prove that a sequence $< \dfrac{2n-7}{3n+2} >$

(a) is monotonically increasing, [4]

(b) is bounded and [4]

(c) tends to the limit $\dfrac{2}{3}$. [4]

(b) If a series $\sum u_n$ is convergent, then $\lim_{n\to\infty} = 0$. Is the converse true? Explain your answer. [8]

Question 4

(a) Suppose $\sum a_n$ and $\sum b_n$ are positive term series with $a_n \leq b_n$ for all n. If $\sum b_n$ converges, show that so does $\sum a_n$. [8]

(b) Let $\sum_{n=1}^{\infty} a_n$ be series of positive terms. Name at least three tests for convergence of this series. [4]

(c) Prove that if $\sum a_n$ is absolutely convergent, then it is convergent (i.e., every absolutely convergent series is convergent). [8]

Question 5

(a) Let $f(x) = \dfrac{x^2+2}{x^2+1}$, then given $\epsilon > 0$, find a real number δ such that $|f(x) - 2| < \epsilon$ whenever $0 < |x| < \delta$. [6]

(b) Show that the function defined by $f(x) = x^2$ is uniformly continuous on $[-2, 2]$.

[6]

(c) Using $\epsilon - \delta$ definition, prove that
$$f(x) = \begin{cases} x \sin \frac{1}{x} & if x \neq 0 \\ 0 & if x = 0 \end{cases}$$ is continuous at $x = 0$ [8]

Question 6

(a) Let $f(x) = x$ for $x \in [0,1]$ and let $P = \{0, \frac{1}{3}, \frac{2}{3}, 1\}$ be a partition of $[0,1]$. Compute $U(P, f)$ and $L(P, f)$. [8]

(b) Using the definition of the Riemann integral show that $\int_1^2 (2x+3) = 6$. [12]

End of Examination Paper