University of Eswatini

Supplimentary Examination, 2018/2019

BASS IV, B.Ed (Sec.) IV, B.Sc IV

Title of Paper : INTRODUCTION TO MATHEMATICS OF FINANCE

Course Number : MAT 442

Time Allowed : Three (3) Hours

Instructions:

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Start each new major question (A1-A5, B1 – B5) on a new page and clearly indicate the question number at the top of the page.
5. You can answer questions in any order.
6. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1.

(a.) Define a probability space. [2 marks]

(b.) Suppose $X, Y : \Omega \to \Re$ are two given price functions and $g : \Re \to \Re$ is a Borel measurable function. Given that $Y = aX^2 + c;\ a, c \in \Re$, show that price Y is H_X-measurable and identify the conditions. [6 marks]

QUESTION A2.

(a.) Define a stochastic process $\{X_t\}_{t\geq 0}$. [2 marks]

(b.) Let $X : \Omega \to \Re$ be a random variable with cumulative distribution function $F(x)$ given by $F(x) = P[X \leq x]$ and $[0 \leq F \leq 1]$ Show that $\lim_{x\to -\infty} F(x) = 0$ and $\lim_{x\to +\infty} F(x) = 1$. [6 marks]

QUESTION A3.

(a.) Define a σ-algebra. [4 marks]

(b.) Given $(\Omega, \Im, P)$ and a random variable X. When is X said to be $\Im$-measurable. [4 marks]

QUESTION A4.

(a.) Define a Brownian Motion and give 2 financial industry-based Brownian processes. [4 marks]

(b.) Given a Brownian price process $B(t)$. Show that the increments $B_{t_1}, (B_{t_2} - B_{t_1}), (B_{t_3} - B_{t_2}), \ldots$ at distinct times $t_i; i = 1, 2, 3, \ldots$ are independent. [4 marks]

QUESTION A5.

(a.) Given a market process $X(t)$ whose changes is described by

$$\frac{dX(t)}{dt} = a(t, X(t)) + b(t, X(t))."noise."$$

where a(.) and b(.) are some given functions. If the noise is $W(t)$, give the basic properties of $W(t)$. [3 marks]

(b.) Given that $W(t) \sim B(t)$, Construct a general solution for $X(t)$. [5 marks]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B1

a.) Define an elementary function. [2 marks]

b.) List three (3) real valued functions that are elementary functions. [5 marks]

c.) State and prove Ito isometry for elementary and bounded function $\phi(t, \omega)$. [13 marks]

QUESTION B2.

a.) Define an Ito process. [3 marks]

b.) List four (4) properties of the Ito integral. [4 marks]

c.) Evaluate the integral $I = \int_0^t s^3 B^3 dB(s)$. [13 marks]

QUESTION B3.

a.) Define an arbitrage market. [3 marks]

b.) Consider the market process $X(t)$ given by

$$dX_1(t) = 3dt + 2dB_1(t)$$

$$dX_2(t) = -2dt + dB_1(t) + 5dB_2(t).$$

Show that a portfolio $\theta(t)$ should be allowed to do business in $X(t)$. [14 marks]

c.) Find the value process $V^{\theta}(t)$ at expiration time $t = T$. [3 marks]

QUESTION B4.

a.) State the Martingale representation theorem. [4 marks]

b.) Use Ito's formula to prove that if

$$dZ(t) = Z(t)\theta(t, \omega)dB(t)$$

then $Z(t)$ is a martingale for all $t \leq T$ provided that $Z(t)\theta_k(t, \omega) \in \nu(0, T) \quad 1 \leq k \leq n$. [16 marks]

QUESTION B5.

a.) Use Ito's formula to write $X(t)$ in the form

$$dX(t) = u(t,\omega)dt + v(t,\omega)dB(t)$$

(i.) $X(t) = tB^2(t)$. [4 marks]

(ii.) $X(t) = 2 + 4t + e^{B(t)}$. [4 marks]

b.) Find the solution to the Ornstein-Uhlenbeck equation

$$dS(t) = 5S(t)dt + 0.22dB(t); \quad S(0) = 10 \; units$$

representing the change in price $S(t)$ of an option trading in an African stock market at time $t \in [0, t]$. [12 marks]

END OF EXAMINATION