University of Swaziland

Final Examination, May 2019

B.Sc IV, BASS IV, BEd IV

Title of Paper : Fluid Mechanics

Course Code : MAT456/M455

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.

   a. **SECTION A(COMPULSORY): 40 MARKS**
   Answer ALL QUESTIONS.

   b. **SECTION B: 60 MARKS**
   Answer ANY THREE questions.
   Submit solutions to ONLY THREE questions in Section B.

2. Each question in Section B is worth 20%.

3. Show all your working.

4. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) Define the following terms:

(i) Viscous fluid, Laminar fluid, steady flow, irrotational flows. [8]

(ii) Describe the Lagrange method of describing fluid motion. [5]

(iii) Write the components of acceleration in cylinderical coordinates (r, θ, z) with velocity components (v_r, v_θ, v_z). [3]

(b) (i) Define streamlines, velocity potential. [4]

(ii) Give an examples of irrotational and rotational flows. [5]

(iii) Find the vorticity components if the velocity disribution of a fluid particle is $\mathbf{q} = \mathbf{i}(Ax^2yt) + \mathbf{j}(By^2zt) + \mathbf{k}(Czt^2)$.
where A, B and C are constants. [5]

(iv) If the velocity $\mathbf{q} = x\mathbf{i} - y\mathbf{j}$, determine the equation of streamlines. [5]

(v) The velocity components for a two dimensional fluid system can be given in the Eulerian system by $u = 2x+2y+3t, v = x+y+\frac{1}{2}t$. Find the displacement of a fluid particle in the Lagrangian system. [5]

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

(a) Suppose fluid particle moves from $P(x, y, z)$ at time t to $Q(x, y, z)$ at time $t + \delta t$. Further suppose $f(x, y, z, t)$ is any function associated with the fluid property. If $q = (u, v, w)$ is velocity of fluid particle at P, show that

$$\frac{D\mathbf{f}}{Dt} = \frac{\partial f}{\partial t} + (\mathbf{q} \cdot \nabla) f.$$

Also indicate material, convective and local derivatives. [10]

(b) Derive the equation of continuity (vector form) using Euler method. [10]

Question 3

(a) The velocity component of the flow in cylindrical polar coordinates are $(r^2 z \cos\theta, rz \sin\theta, z^2 t)$. Determine the components of the acceleration of the fluid particle. [6]

(b) Test whether the motion specified by

$$\mathbf{q} = \frac{k^2(x\mathbf{j} - y\mathbf{i})}{x^2 + y^2} \quad (k = \text{constant}),$$

(i) is a possible motion for an incompressible fluid. [4]

(ii) If (i) is true, determine the equations of the streamlines. [5]

(iii) test the motion is of potential kind and if so determine the velocity potential. [5]

Question 4

(a) The velocity vector in the flow field is given by

$$\mathbf{q} = \mathbf{i}(Az - By) + \mathbf{j}(Bx - Cz) + \mathbf{k}(Cy - Ax)$$

where A, B and C are non zero constants. Determine the equations of the vortex lines. [14]

(b) An engineer is studying how some insects are able to walk on water. A fluid property of importance in this problem is surface tention (σ_s), which has dimensions of force per unit length. Write the dimensions of surface tension in termes of primary dimensions. [6]

Question 5

(a) A garden hose attached with a nozzel is used to fill a 10 *liters* bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit. If it takes 50 sec to fill the bucket with water, determine

(i) the volume and mass flow rates of water through the hose, [8]

(ii) the average velocity of water at the nozzle exit. [8]

(b) Define Newtonian fluid. [4]

Question 6

(a) Write down Navier-Stakes equation for an incompressible model of fluid and give the physical meaning of each terms. [7]

(b) Show that the Bernoulli's equation for steady irrotational motion of an incompressible fluid is given by

$$\tfrac{1}{2}q^2 + V + \tfrac{p}{\rho} = C$$

where V is force potential, p is pressure, ρ density, q is velocity of fluid particle and C is a constant. (Hint: Apply Euler's dynamical equation of motion). [13]

End of Examination Paper